VARIABLES AND EQUALS

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VARIABLES and EQUALS are one and the same concept.

Imagine an equation with a single variable, such as

$$AX + B = C$$

The EQUAL NORMAL FORM is

$$AX + B - C = 0$$

This abstractly reduces further, since the numerical constants will combine:

$$AX + D = 0$$

The VARIABLE NORMAL FORM is

$$X = (C - B)/A$$

or simply

$$X = -D/A$$

That these appear to be different is an artifact of linear notation, reinforced by our narrow training. We are given problems (EQUAL NORMAL FORM, or ENF) in which X is "unknown", and told to move the problem to the solution (VARIABLE NORMAL FORM, or VNF).

Alternatively, we might think in the other direction. When X is known, how might we decompose its value so as to render it (by operations) equal to nothing? It's easy to imagine a *synthetic* curriculum that rewards the exploration of the forms of nothing rather than the old *analytic* curriculum that has us always untangling someone else's confusions.

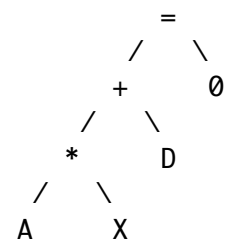
But the key point is that the operations in each approach are identical. Only the direction we are looking changes. This identicality can be easily seen in a two dimensional graph notation

What's coming is similar to Sussman and Steele's famous CONSTRAINTS paper.

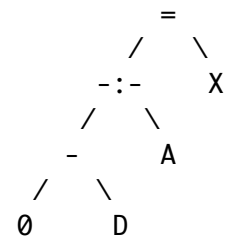
NORMAL FORM TREES

The trees for the two normal forms of the above example are:

Equal Normal Form



Variable Normal Form



The key shift in representation is to eliminate the equal sign. Three definitions:

A *simple* node has only on link.
A *disposable* node has two links.
A *complex* node has three or more links.

Both variables and constants are simple trees, in that they have only one arrow entering or exiting their node.

The ***Disposable Node Principle*** is:

When a node has only two links, the links and the node can be converted into a single link that is defined (labeled) by the node type.

That is, let

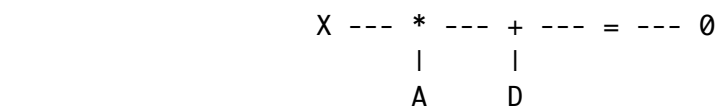
node1 --- node2 --- node3

be represented as

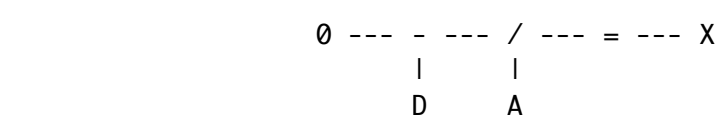
node1 -----n2------ node3

The above two trees are presented next in a flattened version. Only the structure has been changed -- not the organization of the nodes.

Equal Normal Form

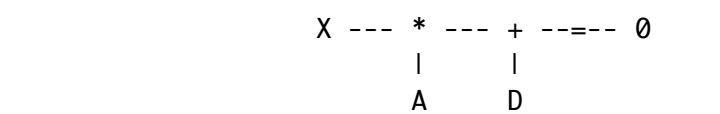


Variable Normal Form

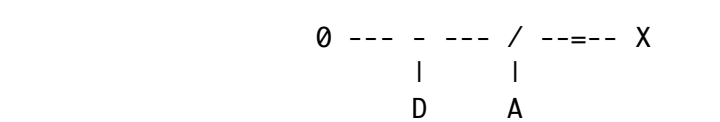


Now we can ELIMINATE THE EQUAL SIGN, since it is a disposable node with only two links. A new type of link is created. (We do not have to be concerned with its semantics here.)

Equal Normal Form



Variable Normal Form



This is just saying that "equal-to-a-form" is the same as "reduces-to-a-form". The each of these two pairs of representations loses no information:

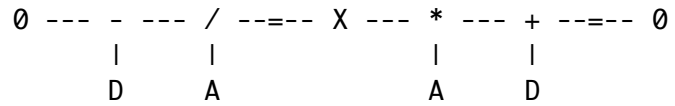
AX + D = 0 and AX + D ==> 0 *ENF*

X = -D/A and -D/A ==> X *VNF*

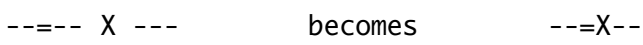
In the above representation, it is easy to see that the two normal forms are similar but for the operators along the top of the representation. Traveling from X to 0 along the ENF is the same as traveling from 0 to X along the VNF, but with inverted operations at each node.

The entire analysis/synthesis cycle is formed by composing the above two representations into a single tree. Composition is simply superimposing an identical node in each tree to form a single tree.

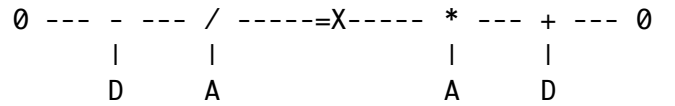
Below, two identical X nodes are combined into one X node.



The VARIABLE can now also be eliminated using the same technique that eliminated the EQUALS. That is, a new arrow is introduced that replaces the combination of two arrows and a node.



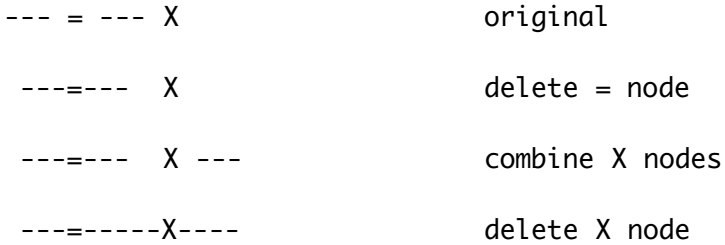
This generates the following tree:



A NETWORK QUERY can be posted at any link. The effect of a query on a link is to split the link into two links and introduce a VARAIBLE node in the middle. That is, querying is simply the inversion of Disposable Node Principle.

The value of the network at a query on a link (what it EQUALS) is the same as the value of a disposable VARIABLE node used to split the link into two.

The interpretation of the network at any link is the same whether the link of interest is labeled with a variable or with an equality. This is visual apparent since the evolution of new links formed by disposing of node is:



Nodes with only two links have something in common, they can be collapsed into a single link labeled by the eliminated node. Both X and = collapse together.
That is, in the right notation:

EQUALS and VARIABLE are the same concept.