History of Logic

Ancient Party Games

Logic has confused, perplexed, and challenged philosophers and scholars from the beginning of culture. It was built into our language (and presumably our thinking) from the beginning of language. However, philosophers did not (and still do not) understand the subtleties of the simple words {true, false, and, or, not, if, equal, some, all, therefore}

Some men are barbarians. Some barbarians are kind. Thus, some men are kind.	Is this a proper conclusion?
If it is raining, then I am happy. If I am dead, then I am happy	Is this necessarily True when I am in the rain? Is this "if" the same as the above "if"?
He or me. Watch or listen.	Are there two types of "or"? (exclusive and inclusive)
If you say that you are lying and that is the truth, then you are lying.	What do paradoxes mean? (Cicero)
Is. Not is. Not not is.	Does “not not” mean nothing at all?

Aristotle

Aristotle was the first person to classify declarative language. He used three polar categories:

single vs compound	Socrates is happy vs Man is happy.
universal vs particular	Everyone vs someone.
affirm vs deny	Everyone vs no one.

The latter two categories form the **Square of Opposition.**

		AFFIRM		*DENY*
UNIVERSAL	A	Every__is__.. (Everyone is happy.)	E	No__is__. (No one is happy.)
PARTICULAR	I	Some__is__. (Someone is happy.)	O	Some__is not__. (Someone is not happy.)

The Syllogism

According to Aristotle, the fundamental unit of reasoning is the *syllogism*. He defined it as

> "discussion in which, when things are posited, other things necessarily follow."

All men are mortal.
Socrates is a man.
Thus, Socrates is mortal.

The syllogistic form of logic was developed into the first ever **Axiomatic System** with variables.

The Figures of the Syllogism

The ancients discovered these four figures or forms of syllogism, which applied no matter what proposition was substituted for each of the three terms. Here,

C stands for the major term,
M stands for the middle term, and
B stands for the minor term of a syllogism.

The figures (or forms) of reasoning:

```
      C _ M       M _ C       C _ M       M _ C
      M _ B       M _ B       B _ M       B _ M
Thus  B _ C       B _ C       B _ C       B _ C
```

The blanks can be any one of the "AEIO" forms from the square of opposition.

```
A:    Every _ is _.
E:    No    _ is _.
I:    Some  _ is _.
O:    Some  _ is not _.
```

Scholastic Logic

The syllogism survived the Dark Ages in the form of the **rules of theological debate**. During the 13th century, Pope John XXI wrote a book on logic which dominated logical thought for the next 300 years. He observed that:

Nouns and *Verbs* form *Subjects* and *Predicates*

These subjects and predicates are CATEGORMATA; they have a **referent** in the real world.

The logical connectives are SYNCATEGOREMATA; they are **without** a referent in the real world.

Theological debates noticed the **use/mention** distinction:

Man is mortal. versus Man is a noun.

and the **paradoxes** generated by the absence of articles in Latin:

The man is mortal. versus Man is mortal.

The Categorical Syllogism Chant

There were only 19 syllogisms (rules of logic) in the middle ages; no one had figured out the mathematical symmetries (or the logic) which generate 24 balanced syllogisms formed by taking three pairs of four things. No one was bothered by the contradictions in the naming scheme either, since logic was to be *memorized* rather than deduced. The vowels in each of the Latin names for the *moods* of the syllogism are a mnemonic for the AEIO form, which had become associated with the logical connectives.

```
A:    ->          (implies)
E:    ->~         (implies not)
I:    &           (and)
O:    &~          (and not)
```

Quantification also began to show up in unprincipled ways (Q in some forms is "there exists").

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Figure I
      Barbara:    M ->  C     and   B ->  M     thus  B ->  C          (AAA)
      Celarent:   M -> ~C     and   B ->  M     thus  B -> ~C          (EAE)
      Darii:      M ->  C     and   B  &  M     thus  B  &  C          (AII)
      Ferio:      M -> ~C     and   B  &  M     thus  B  & ~C          (EIO)

Figure II
      Cesare:     C -> ~M     and   B ->  M     thus  B -> ~C          (EAE)
      Camestres:  C ->  M     and   B -> ~M     thus  B -> ~C          (AEE)
      Festino:    C -> ~M     and   B ->  M     thus  B  & ~C          (EIO)
      Baroco:     C ->  M     and   B -> ~M     thus  B  & ~C          (AOO)

Figure III
      Darapti:    M ->  C     and  QM ->  B     thus  B  &  C          (AAI)
      Disamis:    M  &  C     and   M ->  B     thus  B  &  C          (IAI)
      Datisi:     M ->  C     and   M  &  B     thus  B  &  C          (AII)
      Felapton:   M -> ~C     and  QM ->  B     thus  B  & ~C          (EAO)
      Bocardo:    M  & ~C     and   M ->  B     thus  B  & ~C          (OAO)
      Feriso:     M -> ~C     and   M  &  B     thus  B  & ~C          (EIO)

Figure IV
      Bamalip:    C ->  M     and   M ->  B     thus  B  & QC          (AAI)
      Calemes:    C ->  M     and   M -> ~B     thus  B -> ~C          (AEE)
      Dimatis:    C  &  M     and   M ->  B     thus  B  &  C          (IAI)
      Fesapo:     C -> ~M     and  QM ->  B     thus  B  & ~C          (EAO)
      Fresison:   C -> ~M     and   M  &  B     thus  B  & ~C          (EIO)
```

Meanwhile in the Non-European World

In 10th century Baghdad, the Nestorian **Abu Bishr Matta ibn Yunus** refined Aristotle's logic, but his work was lost in the passage of time.

In India, logic was hotly debated in a form which differed only slightly from the syllogism:

The mountain is fiery	that is the Proposition
Because smoky	that is the Reason
All that is smoky is fiery	that is the Example
So here	that is the Application
Therefore it is so.	that is the Conclusion

The use of *negation* caused debate:

why should the same words in different order have different meanings?

He shall-not look.
He shall not-look.
Not-he shall look.

"Absence of constant absence of pot is essentially identical with pot"
-- Mathuranatha c. 1700

In the West, Logic Evolved into Formal Systems

Renaissance: Logic was ignored (experience was in vogue)

Enlightenment: Leibniz sought a Universal Calculus of Reason, and studied Indistinguishability.

1850 **Boole**: expressed sentences and noun expressions as algebra

`x + y = y + x`	associativity of OR
`x (y + z) = x y + x z`	distribution of AND over OR
`if x = y then x + z = y + z`	algebraic substitution

1880 **Venn**: logical diagrams

1885 **Peirce**: truth tables

1900 **Russell**: logical foundations of mathematics

1920 **Post**: metalogic (just what are we doing?)

Crisis in the Twentieth Century

Oh No! There is **no consistency in mathematics**, there are paradoxes in every system.

Logicism Bertrand Russell

Mathematics is identical to logic.
(We'll patch the holes.)

Intuitionism L.E. Brouwer

Mathematics presupposes concepts.
Concepts rest on natural numbers.
(We'll construct what is known, and not admit infinity.)

Formalism David Hilbert

Mathematics is a set of syntactic transformations.
(We'll refuse to interpret it.)