

## Propositional Logic

### What is Logic?

the laws of thought	(Boole, c1850, mathematics)
principles of right reasoning	(religion)
methodology of valid argumentation	(law)
study of grammar	(linguistics)
stages of cognitive development	(Piaget, developmental psychology)
truths based solely on the meaning of the terms	(math)
the most abstract and general description of reality	(philosophy)
force of reason rather than dogma	(politics)
science or history of the human mind	(Encyclopedia Britannica, 1771)
technique for design of	(computer science)
circuitry	
program control	
process description	
structured programming	
deductive computation	
programming connectivity	
decision making in algorithms	

### The Elements of Programming

"A powerful programming language...serves as a framework within which we organize our ideas about processes."

-- Abelson and Sussman, "Structure and Interpretation of Computer Programs"

### Three Mechanisms

1. *Primitive expressions*  
the simplest entities that the language addresses
2. *Means of combination*  
compound elements are built from simpler ones
3. *Means of abstraction*  
compound elements are named and manipulated as units.

### Representation

**Lexicon:** the typographical forms which represent statements

**Syntax:** the rules of composition, making compound forms out of objects and functions.  
Atomic objects are propositions, functions and relations.  
Sentences are atomic objects + logical connectives.

**Semantics:** the rules of meaning, connecting statements to values

## Boolean Algebra = Propositional Logic

Boolean algebra is the algebraic approach (match and substitute using equations), and Propositional calculus is the logical approach (inference using conjunction of facts)

to the *same mathematical structure*

(even though the fields developed independently, and don't talk to each other)

Both address the **easiest and simplest** useful formal system,  
which poses the **hardest and most important** technical issues for computation.

## Propositional Calculus

The simplest formal system with great utility. A proposition is the simplest complete unit of thought. Any statement or decision with a Yes/No or True/False result.

**Value Domain:** {True, False}

**Object Domain:** statements (propositions) that are either True or False  
{p, q, r, s...}

**Operator Domain:** connectives {not, or, and, if-then, if-and-only-if}

## Constructing Sentences

The **logical connectives** allow construction of compound ideas, which can include several propositions. for example: (if (A and B) then (C or (not D)))

The **truth value** of a compound sentence is the truth value of its component parts.

(A and B)	isTrue exactly when both A isTrue and B isTrue.
(not A)	isTrue exactly when A is False.
(A or B)	isTrue exactly when either A isTrue or B isTrue.
(A implies B)	isTrue exactly when either (not A) isTrue or B isTrue
(A iff B)	isTrue exactly when either A and B are both True or A and B are both False

There are 16 unique Boolean connectives of two variables, but only five are common {and, or, not, if-then, if-and-only-if}. All connectives can be expressed using only one {nor}.

## Tautologies, Contradictions, and Indeterminate Sentences

Sentences that are always true regardless of the values of the atoms are called *tautologies*. A tautology conveys no information about its components.

Sentences that are always false are *contradictions*.

Sentences which do depend on (at least one of) their component atoms are *indeterminate*.