

## DON'T-CARE OPTIMIZATION

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Some paths within a circuit are never traversed. When these happen to be the topologically longest paths, the timing of the circuit is not optimal. Here is an example of the kind of algebraic transformations we are doing to optimize timing by eliminating don't-care paths.

### Example: two-bit carry by-pass adder

The carry-bypass adder is a ripple-carry adder with two extra gates that bypass the ripple-carry chain when the carry bits are 1, thus shortening the most critical path in the circuit.

The circuit algebraically:

inputs:  $a_0$   $a_1$   $b_0$   $b_1$   $c_0$

outputs:  $s_0$   $s_1$   $c_1$

gates are numbered in PUN format:

1 = ( $a_0$ $b_0$ ) ( $(a_0)(b_0)$ )	xor	
2 = $((a_0)(b_0))$	and	
3 = ( $a_1$ $b_1$ ) ( $(a_1)(b_1)$ )	xor	
4 = $((a_1)(b_1))$	and	
5 = ( $c_0$ 1 ) ( $(c_0)(1)$ )	xor	$s_0$
6 = $((c_0)(1))$	and	
7 = 2 6	or	
8 = ( 3 7 ) ( $(3)(7)$ )	xor	$s_1$
9 = $((3)(7))$	and	
10 = $((1)(3))$	and	bypass
11 = 4 9	or	
12 = ( 10 11 ) ( $c_0$ (10) )	mux	$c_1$ bypass

What follows is the Losp reduction and restructuring of this circuit.

## COALESCE

```

A = (a0 b0)          nor
B = (a1 b1)          nor
2 = ((a0)(b0))      and
4 = ((a1)(b1))      and
6 = ((c0)(1))       and

1 = ( A 2 )          xor
3 = ( B 4 )          xor
5 = ( (c0 1) 6 )     xor      s0
7 = 2 6              or
8 = ( (3 7) ((3)(7)) ) xor      s1
9 = ((3)(7))        and
10= ((1)(3))        and      bypass
11= 4 9              or
12= ( (10 11) (c0 (10)) ) mux    c1  bypass

```

## EXPAND (substitute 1,3,7,9,11)

```

A = (a0 b0)          nor
B = (a1 b1)          nor
2 = ((a0)(b0))      and
4 = ((a1)(b1))      and
6 = ((c0) A 2)       and

5 = ( (c0 (A 2)) 6 ) xor      s0
8 = ( ((B 4) 2 6) (B 4 (2 6)) ) xor    s1
10= (A 2 B 4)        and      bypass
12= ((10 4 (B 4 (2 6))) (c0 (10))) mux  c1  bypass

```

## EXPAND (substitute 6,10)

```

A = (a0 b0)          nor
B = (a1 b1)          nor
2 = ((a0)(b0))      and
4 = ((a1)(b1))      and

5 = ( (c0 (A 2)) ((c0) A 2) )          s0
8 = ( ((B 4) 2 ((c0) A 2)) (B 4 (2 ((c0) A 2))) ) s1
12= (((A 2 B 4) 4 (B 4 (2 ((c0) A 2)))) (c0 A 2 B 4)) c1

```

SIMPLIFY (8,12)

$$8 = ((B \ 4) \ 2 \ ((c0) \ A \ 2)) \ ( \ B \ 4 \ (2 \ ((c0) \ A \ 2))) \ ) \quad s1$$

$$8 = ((B \ 4) \ 2 \ ((c0) \ A \ )) \ ( \ B \ 4 \ (2 \ ((c0) \ A \ ))) \ ) \quad s1$$

$$12 = (((A \ 2 \ B \ 4) \ 4 \ (B \ 4 \ (2 \ ((c0) \ A \ 2)))) \ (c0 \ A \ 2 \ B \ 4)) \quad c1$$

$$12 = (((A \ 2 \ B \ ) \ 4 \ (B \ (2 \ ((c0) \ A \ ))) \ (c0 \ A \ 2 \ B \ 4)) \quad c1$$

COALESCE

$$A = (a0 \ b0) \quad \text{nor}$$

$$B = (a1 \ b1) \quad \text{nor}$$

$$2 = ((a0)(b0)) \quad \text{and}$$

$$4 = ((a1)(b1)) \quad \text{and}$$

$$C = (A \ 2) \quad \text{nor}$$

$$D = (A \ (c0)) \quad \text{nor}$$

$$5 = ( \ (c0 \ C) \ ((c0) \ (C)) \ ) \quad s0$$

$$8 = ( ((B \ 4) \ 2 \ D) \ ( \ B \ 4 \ (2 \ D)) \ ) \quad s1$$

$$12 = (((C) \ B) \ 4 \ (B \ (2 \ D))) \ (c0 \ (C) \ B \ 4)) \quad c1$$

COALESCE

$$A = (a0 \ b0) \quad \text{nor}$$

$$B = (a1 \ b1) \quad \text{nor}$$

$$2 = ((a0)(b0)) \quad \text{and}$$

$$4 = ((a1)(b1)) \quad \text{and}$$

$$C = (A \ 2) \quad \text{nor}$$

$$D = (A \ (c0)) \quad \text{nor}$$

$$E = (B \ 4) \quad \text{nor}$$

$$F = (D \ 2) \quad \text{nor}$$

$$5 = ( \ (c0 \ C) \ ((c0) \ (C)) \ ) \quad s0$$

$$8 = ( (E \ (F)) \ ((E) \ F) \ ) \quad s1$$

$$12 = (((C) \ B) \ 4 \ (B \ F)) \ (c0 \ (C) \ (E))) \quad c1$$

FLEX 8

$$8 = (E \ F) \ ((E)(F))$$

## FACTOR 12

$$\begin{aligned}
 12 &= (((C) B) 4 (B F)) (c0 (C) B 4)) \\
 &= 4 ( (((C) B) (B F)) (c0 (C) B) ) \\
 &= 4 ( B (C (F)) (c0 (C) B) ) \\
 &= 4 ( B (C (F)) (c0 (C) ) ) \\
 &= 4 ( B (C 2 D) (c0 (C) ) ) \\
 \\ 
 &= 4 (B ((A 2) 2 (A (c0))) (c0 A 2) ) \\
 &= 4 (B ((A ) 2 (A (c0))) (c0 A 2) ) \\
 &= 4 (B ((A ) 2 ) (c0 A 2) ) \\
 &= 4 (B (2 (A (c0 A))) ) \\
 &= 4 (B (2 (A (c0 ))) ) \\
 &= 4 (B (2 D) ) \\
 &= 4 (B F)
 \end{aligned}$$

## READING THE RESULT

A = (a0 b0)	nor
B = (a1 b1)	nor
2 = ((a0)(b0))	and
4 = ((a1)(b1))	and
C = (A 2)	nor
D = (A (c0))	nor
E = (B 4)	nor
F = (D 2)	nor

5 = ( (c0 C) ((c0)(C)) )	s0
8 = (E F) ((E)(F))	s1
12= 4 (B F)	c1

A & 2:	a0 = b0
B & 4:	a1 = b1
C:	a0 ≠ b0
E:	a1 ≠ b1
5:	c0 ≠ C
8:	E = F

c1 = 4 (B (2 (A (c0))))      hierarchical dependencies