

COUNTER-EXAMPLE GENERATOR

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Erase all atoms in the outermost space of representation. Erase all duplicates of these atoms throughout the expression.

Erase all bounded atoms -- eg: (a) -- and their boundaries in the outermost space. Substitute () for all duplicates of the bounded atoms throughout the expression.

CLEAN -- ie, ABSORB and CLARIFY) the remaining expression.

Recur to Step 1 if more atoms or bounded atoms occur in the outermost space as a result of simplification.

If () remains, conclude that a tautology is unavoidable. If *nothing* remains, a counterexample has been identified.

If bounded expressions remain, for example

$$(a\ b)\ (a\ (b))\ (b\ (a))\ ((a)(b))$$

which represents

$$a = b\ \text{OR}\ a \neq b$$

then attempt to make every expression disappear. Select one subexpression.

Step 1. If atoms exist at the top level of the subexpression, choose one and substitute () for it throughout the entire expression. If the entire expression vanishes, a counterexample has been found. If the entire expression reduces to (), then the selected atom cannot generate a counterexample. Erase it from the entire expression. Recur to Step 1 until a counterexample is found or until there are no more atoms at the top level of the subexpression.

Step 2. If bounded atoms exist at the top level of the subexpression, choose one and erase it throughout the expression. If the entire expression vanishes, a counterexample has been found. If the entire expression reduces to (), then substitute () for the bounded atom throughout the expression. Recur to Step 1.

Step 3. Otherwise recur to Step 1 using the subexpression as the target expression.