SYNOPSIS OF TECHNICAL COMPONENTS OF GRANT PROPOSAL TO AMERICAN HONDA FOUNDATION

============ Long-term goals ============

Our goal is to provide empowering learning tools for students who are math challenged, including visual thinkers and those who are learning disabled or otherwise disadvantaged. In the long-term, we hope to improve math performance in the State of Washington, and possibly nation-wide. The vision and potential impact of this Project is indeed broad: we will be building curriculum tools for a new conceptualization of arithmetic that has a potential to make understanding and using numbers simpler, an event that has occurred only a few times in the evolution of mathematics. To our knowledge, curriculum materials that teach this powerful technique do not yet exist.

Attachment Q11 [INCLUDED BELOW] describes the Spatial Arithmetic Project we seek to fund, and provides an historical context for the innovation. A CD labeled "Spatial Arithmetic Curriculum Materials Demo Disk" is also included in Attachment Q11. The disk contains a ten minute narrated prototype animation of Spatial Arithmetic used for addition, subtraction, multiplication and division. A shortened three minute version is also on the disk.

============ Short-term objectives ============

Our immediate objective is to develop and informally evaluate new curriculum materials for Spatial Arithmetic. The Project will develop 1) high quality curriculum materials for Spatial Arithmetic, and 2) a public web-site for dissemination of these materials. These products are tangible results in themselves.

Students with concrete, visual or tactile learning styles may benefit directly from the proposed curriculum materials, as would students who are math challenged, disabled, or underprivileged. Performance improvement by students using the new curriculum materials will provide a measure of both prospect and impact. Comparative performance with and without the supplemental Spatial Arithmetic curriculum will provide statistical verification of potential.

Approximately 550 students per year enroll in LWTC Basic Math courses; several times that enroll in other math courses. Successful curriculum materials can be in use for many years without high revision costs, and can be generalized to other math courses. Assuming the program cost to be $80,000, the prorated per student cost for improving learning experiences in Basic Math alone is approximately $24/student for serving over 3300 students. The unit-of-service costs again come down remarkably when successful curriculum materials are widely distributed, potentially to hundreds of thousands of students.

============ Specific Activities ============

We propose to design and develop interactive, animated curriculum materials, and use these materials in classrooms to informally assess their potential. Attachment Q13 [INCLUDED BELOW] includes:

1) A listing of Project tasks, with a timeline and person-hours level-of-effort for Senior and Junior personnel.

2) A listing of design, development, implementation and evaluation tasks by Quarter (for four Quarters).

3) A listing of the envisioned curriculum materials to be developed.

============ Data Collection and Evaluation Procedures ============

The primary purpose of evaluation of prototype educational materials is to informally determine:

-- whether or not the innovative techniques have potential for enhancing understanding of curriculum topics,

-- which target populations are most likely to be helped, and

-- whether or not the initial prototype designs can be refined into useful classroom tools for wider distribution.

We will seek to answer these two questions: Do the materials work as expected? How can they be improved?

For Development Iteration I, we will replicate a selection of units from our current curriculum for Basic Mathematics, providing supplemental learning experiences using conventional and innovative methods. The curriculum evaluation will include soliciting student and instructor opinions, formative evaluations, and debugging comments.

For Development Iteration II, we will refine both software and curriculum to reflect lessons learned. Field testing will include observation of users, protocol analysis of user interaction, informal interviews, and structured questionnaires. Evaluation will provide pilot data for performance, but the main focus will still be on formative evaluation through discussion and debugging with students and faculty.

For Development Iteration III, we will compare performance in classrooms using the innovative tools to performance in classrooms not using the curriculum tools.

We will prepare

-- a critical analysis of the utility of the new tools,

-- a prospectus of future development, and
-- a web site for dissemination of results and materials

as informed by our experiences throughout the Project year.

============ Pros and Cons ============

PROS FOR FUNDING THE DEVELOPMENT OF SPATIAL ARITHMETIC CURRICULUM MATERIALS

1. Demonstration that basic mathematics can be taught with rigor by visual, interactive and experiential techniques that are inherently easier to understand and to use than conventional symbolic techniques may have broad impact. There is an explicit need for improved math curriculum materials in high schools, since 49% of tenth-graders and 75% of minority students failed the math portion of the 2006 Washington Assessment of Student Learning (WASL) tests. The urgent need for innovative tools in math education in Washington State, and nation-wide, suggests a high potential for success, possibly contributing to the advancement of science.

2. Students who are disabled, underprivileged, and/or math challenged and students whose cognitive style is concrete, visual or tactile, deserve and often require non-symbolic approaches to mathematics. Developing new techniques for understanding and for using basic mathematics is humanistic and fulfilling.

3. Deployment of innovative math education pilot curriculum materials that are unprecedented (untried by any educational community) is imaginative and creative.

CONS FOR FUNDING THE DEVELOPMENT OF SPATIAL ARITHMETIC CURRICULUM MATERIALS

1. Although mathematically rigorous, these innovative tools and materials are foreign to established mathematics teaching methods which focus almost exclusively on symbolic techniques and on abstract symbol manipulation. For this reason, we must introduce them as supplemental until they are fully evaluated.

2. The concrete, visual, tactile and experiential learning tools of Spatial Arithmetic require high technology support for display and use.

3. There is a risk that spatial and symbolic techniques may get confused in a student's mind.

ATTACHMENT Q11

LONG-TERM GOALS OF THE PROPOSED "SPATIAL ARITHMETIC" PROJECT

This attachment includes three items:

1. A brief description of the proposed Spatial Arithmetic Project.

2. "Evolution and Growth of Arithmetic" (3 pages), which places the mathematics of this project in an historical context of innovations in writing and transforming numbers.

3. The "Spatial Arithmetic Curriculum Materials Demo Disk", includes:
   A. An eleven-minute narrated demonstration of Spatial Arithmetic, showing addition, subtraction, multiplication, and division.
   B. A condensed four-minute version extracted from Item A.

[NOT INCLUDED HEREIN]

THE SPATIAL ARITHMETIC PROJECT

We propose to develop curriculum materials for Spatial Arithmetic, a recent innovation in elementary mathematics. We intend to use these tools at LWTC as a supplement to math courses designed for students in our automotive repair, electronics, equipment service, and other applied technology programs.

This work is not the product of university research departments, and has thus far been developed without funding support. With funding, we hope to be able to take the first forward-looking steps: to show the possibility and the potential of connecting symbolic mathematics to direct experience. Even with small whole numbers, children can solve problems through direct manipulation of objects that they cannot solve when these problems are expressed in symbolic form. Our work may provide a bridge between interactive mathematics and abstract symbolism.

Spatial notations such as those used in graphing and in geometry provide unique visualization experiences for learners. The visual and interactive numbers of Spatial Arithmetic support exceedingly easy rules for addition, subtraction, multiplication and division. We intend to explore whether or not a visual and manipulable format will be of advantage to students who need a stronger grounding in how and why mathematics works.

The Mathematics and Technical faculties at Lake Washington Technical College embrace Honda's vision. Our proposed Project demonstrates a completely new way to understand elementary mathematics, one that is untried but with solid formal underpinnings that can lead to rigorous scientific growth. We hope that The

American Honda Foundation will share the courage and excitement of taking the very first steps to explore the tools and techniques of Spatial Arithmetic.

EVOLUTION AND GROWTH OF ARITHMETIC

Numbers can be written and transformed -- by addition, subtraction, multiplication and division -- in a variety of ways. There is a trade-off between the difficulty of reading a number (its representation) and the difficulty of transforming it (its operations).

Current place-value arithmetic is easy to read, but somewhat clumsy to transform. Spatial Arithmetic is easy to read, and it is also easy to transform. Here, we illustrate the potential impact of this work by placing it in an historical context. We show how addition works for early stroke-numbers, for Roman numerals, for our current place-value notation, and for the innovative depth-value notation of Spatial Arithmetic. We also show how multiplication works, for the very inconvenient Roman numerals, for the somewhat convenient place-value notation, and for the very simple multiplication of depth-value notation.

STROKE-NUMBERS

The earliest arithmetic was unit-based, sometimes called stroke or tally arithmetic.

17: /////////////////

Strokes map one-to-one to a collection of objects, such as sheep or buckets of water. Stroke arithmetic makes addition simple; to add two stroke-numbers, you simply place them together in a shared space:

5 + 7 = 12: ///// /////// ==> ////////////

Stroke-numbers, however, are difficult to read. It is necessary to count the combined pile to read the sum. Stroke-numbers are fairly easy to multiply, by replicating the number being multiplied once for each stroke in the base number. Again reading turns out to be particularly difficult.

ROMAN NUMERALS

The Romans introduced names for collections of a specific numerical size. For example, what we call "five" was spatially converted to the shape "V".

5: ///// = V

The problem of readability in stroke-numbers is addressed by having names for large groups, such as M for 1000. Like stroke-numbers, Roman numerals can still be added by being placed together in a shared space:

1171 + 2087: MCLXXI MMLXXXVII ==> MMM C LL XXXXX V III

The result of a sum is significantly easier to read. Specific "Roman-number facts", such as

5 + 5 = 10: V V = X

improve the ease of reading even more:

3258: MMM C LL XXXXX V III ==> MMM CC L V III

Multiplication of Roman-numerals is very difficult, in fact multiplication was a university graduate school topic in the fifteenth century.

TODAY'S ARABIC NUMBERS

The Arabic decimal system introduces a common base (i.e. 10). Individual number-names are needed only for the digits 0 through 9, and not for larger groups. The simple shorthand notation that we use today, place-value notation, relies upon increasing multiples of the base 10 to express larger groups:

3258: (3x1000) + (2x100) + (5x10) + (8x1)

Arabic-numbers exchange a great gain in readability for a moderate loss in computability. Numbers no longer add by mere spatial combination, instead they require both number facts (such as 4 + 5 = 9) and tracking the place-value of each digit. Digits are maintained in a strict sequential position; calculation then includes techniques for interfacing adjacent places, called "carrying" and "borrowing". Place-value numbers are multiplied using the techniques of addition combined with new number facts (the multiplication table), making it possible for us today to introduce multiplication in third grade.

BOUNDARY-NUMBERS

Boundary-numbers are spatial pictures rather than strings of digits. The techniques were first published in 1995 by Louis H. Kauffman, a Full Professor of Mathematics at the University of Illinois at Chicago. Spatial Arithmetic is dynamic, it shows the structure and transformation of numbers directly without the bookkeeping associated with place-values. As an analogy, if the techniques of Spatial Arithmetic applied to words, we would be able to read a word regardless of the ordering of the letters!

Please refer to the animation for a visualization of Spatial Arithmetic. The accompanying text also demonstrates a convenient notation that permits recording boundary-numbers on typographical lines.

In Spatial Arithmetic, addition is achieved by placing two or more boundary-numbers in the same space, just like stroke arithmetic and Roman numerals. Unlike stroke arithmetic, depth-value notation provides the same advantages in readability as does place-value notation.

Instead of addition operations, boundary-numbers are simplified by a standardization process ("boundary-number facts") that results in a form representing the sum. This standardization process is extremely simple, consisting of two rules (here shown using a binary base for clearer presentation):

1 + 1 = 2:	• • = (•)
2 + 2 = 2(1+1):	(•)(•) = (• •)

Boundary-numbers are just as simple to multiply. Multiplication is achieved by substituting the number being multiplied for each unit (i.e. each •) in the base number. This substitution alone achieves multiplication. The same standardization process, using the same two simple rules above, converts the result of multiplication into an easier to read form. Subtraction and division are also achieved using the same two rules, applying them in reverse, from right to left.

Decimal numbers (base 10) are easily accommodated by the methods of Spatial Arithmetic. Each boundary multiplies its contents by 10 rather than by 2 as is the binary case. Simple number facts are necessary for adding and multiplying digits between 0 and 9. The fundamental transformation principles of addition by sharing a space, and multiplication by substitution still apply. An example of a base-10 boundary-number:

3258:	((((3) 2) 5) 8

We hope that The American Honda Foundation will be able to see directly, from the animation, some of the inherent advantages of Spatial Arithmetic. It is formal and rigorous, while at the same time it is fundamentally visual, interactive, and simple. Spatial Arithmetic has the potential to greatly reduce the difficulty of computation in arithmetic, leading to a greater ease of acceptance and understanding by students who have difficulty with conventional symbolic mathematics.

ATTACHMENT Q13

SPECIFIC ACTIVITIES

The Project tasks, with a timeline and person-hours effort for Senior and Junior personnel follows. NOTE: This schedule assumes a start date in Spring, with much of the development work done over the Summer.

```
2007                 A   M   J   J   A   S   O   N   D   J   F   M      person
PROJECT MONTH        1   2   3   4   5   6   7   8   9  10  11  12         hours
                                                                        Sr    Jr
Infrastructure       -----                                              40
Design
  math units         ---------   ---------                              80   120
  animation          ---------   ---------------------                  40   720
  interface              -----------------       ---------              40    80
  interaction            -----------------       ---------              80    80
Implementation
  iteration I            -----------------                              40   320
    evaluation I                 ---------                              40   120
    re-engineering               -------------                          40   320
  iteration II                       -----------------                  40   240
    field test                               ---------                  40   120
    evaluation II                                ---------              40
    refinement                                   ---------              40   120
  iteration III                                      ---------          40   240
    classroom test                                   -------------      40   120
    evaluation                                           ---------      40    80
  future work                                                ----       40
Web-site
  development                    -------------------------    ----      80   360
  testing                            -----------------------------           160
  refinement                         -----------------------------      40    80

MONTH                  1   2   3   4   5   6   7   8   9  10  11  12

person-hours Sr       40  40 120 120  80  80  40  40  80  40  40  80
person-hours Jr        0 160 240 400 400 400 240 240 320 160 160  80
TOTAL PERSON-HOURS                                                    800  2800

Reporting
  Quarter I                        x
  Quarter II                                   x
  Quarter III                                              x
  Quarter IV                                                       x
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DEVELOPMENT TASKS

QUARTER I, MONTHS 1-3:

Establish Project and team
- enlist graphics arts and programming students
- enlist instructors and identify classes for testing curriculum
- commit to platforms, presentation software, and animation software
- assemble animation infrastructure
- commit to specific curriculum units

Design
- curriculum elements and units
- tools for building curriculum materials
- interface to curriculum materials
- review math learning problems for target students
- assess and refine levels-of-effort
- rough-out animation, interface, and interaction models

Implementation
- begin Iteration I

Quarter I report

QUARTER II, MONTHS 4-6:

Design
- review first-pass curriculum with target students
- finish curriculum units design
- integrate curriculum design with animations

Implementation
- complete Iteration I
- develop prototype interface
- begin Iteration II
- begin animation and interaction integration
- begin problem presentation tool
- full-documentation of Iteration I
- web-site rough-out

Evaluation
- review all design and development commitments
- informal evaluation of Iteration I

Quarter II report

QUARTER III, MONTHS 7-9:

Design
- complete design evaluation and refinement
- finish interface and interaction design
- formulate final design based on Iteration II evaluations

Implementation
- complete Iteration II
- begin Iteration III
- continue web-site development
- rough-out user manuals

Evaluation
- field test Iteration II
- collect interaction protocols from target students
- informal interface and usability studies
- collect comments and suggestions from math staff

Quarter III report

QUARTER IV, MONTHS 10-12:

Implementation
- complete and polish Iteration III
- complete web-site development
- install all Project materials on web-site

Evaluation
- test materials in classroom setting
- summary evaluation of curriculum materials
- summary evaluation of Project strengths and weaknesses
- assemble and document lessons learned
- development long-term plan for growth

Final report to The American Honda Foundation, Torrance CA

ENVISIONED CURRICULUM MATERIALS

- cover the core of basic mathematics
- units for addition, subtraction, multiplication, and division of
  - whole numbers
  - signed numbers
  - fractions
  - exponents
  - some aspects of simple algebra
- both decimal and binary versions
- teaching units to show
  - relationship between Spatial Arithmetic and properties of numbers
    - identity, zeros, commutativity, associativity, distribution
  - how to read and write boundary numbers
  - the relationship between depth-value and place-value notation
  - operations using Spatial Arithmetic
- interactive display for showing Spatial Arithmetic in use
  - type in conventional problem
  - watch solution unfold using Spatial Arithmetic
  - an Spatial Arithmetic calculator
  - drag-and-drop manipulation of spatial forms
- problem presentation tool
  - pretest and posttest for all unit topics
  - a substantive body of practice problems
    - parallel to conventional text
  - show conventional and Spatial Arithmetic solutions
    - with mapping between components