

REPORT ON MATHEMATICA (ALPHA-VERSION)

Bruce Sawhill
April 27, 1988

In the course of a ten day trial period, I have become acquainted with many of the features and capabilities of Mathematica. I believe this represents a very significant, although not revolutionary, step in the evolution of symbolic manipulation programs. I see Mathematica as a synthesis of many capabilities developed over the last ten years, both in the field of symbolic manipulation and in interface, such as graphics and text processing. With proper nurturing, modification, and distribution, it has a very good probability of becoming a standard of industry and academia over the next ten to fifteen years. It has some glaring problems as well, most of which stem from the fact that the version I have tested is far from complete, making the system intellectually and academically compelling but as of yet not marketable to a wide audience.

I see this system as the symbolic manipulation equivalent of Donald Knuth's TeX. One has a great degree of control over the smallest functionalities of the system, analogous to MetaFont. And like TeX, routines can be nested within routines to higher and higher levels of abstraction and complexity, producing a program architecture with a logical structure that is very transparent and aesthetically pleasing. The system is open ended in terms of complexity, and it would take a considerable amount of time to become truly proficient in all of its capabilities, but it is also very rewarding to the complete neophyte in that commands are intuitive and notation elegant and straight-forward. Like TeX, its full set of capabilities will probably prove to be unwieldy for the average user, and "Macros" will be written by users to streamline special purpose applications.

In practical terms, the system is well thought out and boasts several features that could vastly increase the presence of symbolic computing in many fields. The system integrates graphics and computation in a very efficient and simple way, a hitherto unachieved feat of symbolic manipulation programs. Furthermore, it produces output in PostScript or TeX compatible form, making it remarkably easy to compose publishable documents with excellent graphics and text while simultaneously manipulating and graphing functions in Mathematica. It seems ideally suited to any window-oriented editing system. It is capable of running on any system with a C compiler and 2 Meg of RAM, which puts it in the high end of personal computers. Given that RAM increases exponentially with a 2-folding time of about 18 months, it would seem that Mathematica will have an enormous potential audience within three years.

Technically, the system is very elegant and efficiently designed, producing a remarkable improvement in the area of computation speed, an area

that has traditionally plagued older symbolic manipulation programs such as REDUCE and MACSYMA. In many areas of functionality, Mathematica uses nested algorithms and decision trees to manipulate expressions rather than browsing through enormous look-up tables, as was often the case with MACSYMA. This produces a system that runs much faster, but it also has the disadvantage, for instance, that there are many integrals that it cannot do that MACSYMA could do. Mathematica also has the capability of "dynamic programming", where look-up tables are created by recursive functions to vastly accelerate the evaluation time. It seems to me that this program is part of a natural evolution dictated by technology as it is designed to take full advantage of a large accessible RAM, something that was not so readily available in the late 70's when MACSYMA was being developed.

As a theoretical physicist evaluating the high-end capabilities of Mathematica, I found much to be pleased about and a few glaring shortfalls. Every sophisticated function in the CRC handbook is to be found, as well as some very useful routines such as one that approximates the contribution from the converging tail of an infinite series and a whole group of set theory commands. Mathematica will also be very useful in the scientific community because of its ability to produce output compatible with C and Fortran and to be able to call programs written in these languages from inside a Mathematica session. Integrating number-crunching, curve-fitting, and graphics would save a large amount of wasted time. On the negative side, Mathematica has no capability whatsoever to solve differential equations, a very essential capability of any symbolic manipulation system. This capability can certainly be added to a level never before attained, but it constitutes a large amount of work to do it thoroughly, probably on the order of a man-year.

The system as it stands now is very definitely preliminary. When operating in a server mode, there does not seem to be any error checking, so small glitches such as cable noise can cause the whole session to crash. When it becomes necessary to abort a long calculation, one is often not returned to the top level, hence requiring the starting of a new session. This is particularly irritating when one has defined a lot of functions and functional relations, as they are consequently lost. A combination advantage/disadvantage of the system is the handling of formal infinities. It is possible to calculate integrals with infinity as a limit or infinity as a result. Unfortunately, the implementation of this is not consistent, as there exist some integrals which should give well defined finite answers and instead give "Indeterminate". It seems strange to me that the effort was made to include the notion of real and complex infinity, but that this effort was not continued to include aspects of complex analysis.

The main facility of immediate use for CAD is the integration of functional manipulation and 3-D graphics, making it very easy to produce and view structures in 3-space. It would be possible to create a very powerful tool for manipulating graphic objects by combining Mathematica's graphics

capabilities, list processing structure, and mathematical manipulation features to produce a meta-language that could formally and precisely understand such concepts as parallel, tangent, etc., and translate them into graphics.

On a more sophisticated level, Mathematica could perform stress or thermal analysis on a given drawn structure, then use interpretive graphics to display the result.

In conclusion, I believe that Mathematica constitutes the groundwork for a standardization of computational symbolic mathematics and its integration with text and image processing. There remains a large amount of work to be done, but the structure of the system creates an opportunity for virtually unlimited elaboration and sophistication. This system will become the computing industry standard whether or not we choose to participate, so in the light of such manifest destiny it would seem appropriate that we weight its options with respect to this computing system carefully.