DESIGNING VIRTUAL WORLDS FOR USE IN MATHEMATICS EDUCATION

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Introduction

This paper discusses the use of Virtual Reality (VR) to help students learn school subjects, in this case elementary algebra. Our view is that previous attempts to use innovative technologies in classrooms have not been as successful as they might have been because fundamental assumptions about how students learn and about how technology might best represent course content have been addressed as afterthoughts, if at all. Our project is concerned first and foremost with applying current research on learning to improve students' classroom experiences. This approach has allowed us to preserve the benefits to students that arise from judicious application of learning theory and from the unique system that one of us (Bricken) has developed to represent part of the Algebra curriculum. In short, our approach has emphasized responsible design and has led us to interesting innovations in learning strategy and in the representation of the content of mathematics.

Virtual reality is a computer generated, multi-dimensional, inclusive environment which can be accepted by a participant as cognitively valid (W. Bricken, 1992). VR presentation systems overcome the inconvenience of an insufficiently abstract physical reality by combining mathematical abstraction with the intuition of natural behavior. The programmability of VR allows a curriculum designer to embed pedagogical strategies into the behavior of virtual objects which represent mathematical structures (M. Bricken, 1991b). Using a VR presentation system, the axioms of algebra can be, so to speak, built into the behavior of the world.

This paper is a report of work in progress. We have thus far focused on experimental and representational design. During the next phase of our research, we will observe the use of VR in ninth grade Algebra classrooms.

The report first addresses learning theory. We believe that students learn best when they construct understanding for themselves. Our approach therefore has an obviously constructivist flavor. We then address knowledge representation. The symbolic system used to represent the concepts and procedures of Algebra in traditional approaches to Mathematics instruction is textual, somewhat arbitrary, and difficult for many students to master. This impedes their understanding of essential fundamental algebraic concepts. The Spatial Algebra we describe represents a totally new way of representing algebraic concepts and procedures. We believe that it removes the impediments imposed by the traditional text-based symbol system. We conclude with some arguments for the use of VR in classrooms.

Knowledge Construction: A Basis for Learning in VR

In the past, instructional designers have put a great deal of effort into getting technologies to do what teachers do. The assumption has been that if pre-designed instruction can use the same strategies that teachers use, with similar or better results, then the designer has been successful. Instructional systems have therefore been didactic and based upon reductionistic and deterministic theories of learning, instructional design and teaching. There is growing opinion and evidence, however, that these theories and the instructional models that arise from them are severely constrained in their ability to explain human learning and to guide effective instruction (Bednar, Cunningham, Duffy & Perry, 1991; Streibel, 1989; Winn, 1990).

Recently, educational technologists have begun to explore alternatives to didactic, deterministic systems that teach particular content, such as CAI and intelligent tutors. These alternatives are shells that facilitate certain pedagogical strategies without specifying content. Zucchermaglia (1991) appropriately dubs these empty as opposed to filled technologies. These systems embody the idea that technologies employ symbol systems that engage cognitive processes in unique ways (Salomon, 1979). Once these aspects of technologies become the focus of designers' attention, they can become "tools for thought" (Salomon, 1988; Salomon, Perkins & Globerson, 1991).

Shell systems are based on the premise that students construct their own meaning by interacting with material rather than being taught something expicitly (Bransford, Sherwood, Hasselbring, Kinzer & Williams, 1990; Cognition and Technology Group, 1991; McMahon & O'Neil, 1991; Scardamalia, 1991; Spiro & Jehng, 1990; Spiro, Feltovich, Jacobson & Coulson, 1991). Constructive techniques are exemplified by the hypermedia system developed by Spiro and his colleagues (Spiro, Feltovich, Jacobson & Coulson, 1991; Spiro & Jehng, 1990) that lets students learn problem-solving through the exploration of ill-structured domains such as literary criticism, military strategy, and cardio-vascular medicine; and by the interactive videodisk materials developed by Bransford and the Cognition and Technology Group at Vanderbilt University (1990, 1991) that facilitate the solving of complex Mathematics problems by allowing children to interact with dramatically presented adventures.

Algebra is different from the content embodied in the systems of Spiro, and Bransford. While Algebra is certainly challenging to students when they begin to study it, it is not ill-structured in Spiro's sense. For this reason, there are "right answers" that students should arrive at and procedures that they should follow. The virtual Algebra world can therefore be designed to guide students in their knowledge construction somewhat more extensively than in the systems we have just described.

Unlike the real world, a virtual world can be programmed by the designer, or by the student, to behave in a specified manner. For example, objects in the Algebra world could obey the laws of Algebra, not those of Newtonian Physics. When a student lets go of a virtual object that represents a term in an equation, it could fall into the proper place in the equation, rather than falling to the ground. A student's understanding of Algebra could be guided by the ways in which the rules of Algebra are programmed to act in the virtual world. For example, if a student fails to change the sign of a term as it moves from one side of an equation to the other, the rules of Algebra might be programmed to apply in one of three ways:

(1) The term could "float back" to where it came from, indicating that the student had made a mistake without revealing what the mistake was;

(2) the sign of the term could be changed by the program, indicating that a mistake had occurred, what it was, and what the correct transformation is; or

(3) the program could allow the student to make the mistake without correcting it on the assumption that ultimate failure to solve the equation would lead the student to "debug" what had occurred.

In this way, guidance can be seamlessly woven into the virtual world simply by varying the degree to which the laws of Algebra, govern behavior of objects in the virtual world. Students therefore would learn to construct their knowledge of algebra from the way in which the virtual world is programmed to act.

The Symbol Systems of Algebra

The difficulties children have when they begin to learn algebra are well documented (Bricken, 1987; O'Shea, 1986; Zehavi & Bruckheimer, 1984; Gerace & Mestre, 1982; Sleeman, 1984). Frequently, these difficulties arise from the novelty and abstruse nature of Algebra's symbol system. Thwaites (1982) found that students are often baffled by algebra's non-visual nature, its apparent arbitrariness, its complexity, and how problems are expressed using its symbols.

If students fail to understand algebraic representation, then the only way they can solve algebra problems is by the rote application of procedures they have memorized. This memorization is brittle, often both over and under generalized, and elaborated by motivations independent of the content of Algebra (Bricken, 1987). Gerace & Mestre (1982) reported that the students they interviewed did not use proper algebraic techniques to solve problems and treated algebra as a rule-based rather than as a concept- based discipline. Unfortunately, the rule-based approach to Algebra is the one that is often taught, even though this does not promote the development of good conceptual models (Thwaites, 1982; Bright, 1981; Bernard & Bright, 1982; Greeno, 1985).

Since the symbol system of algebra is a major stumbling block to the development of conceptual models, it is not surprising that a number of attempts to help beginning students overcome their difficulties have focused on helping them understand the algebraic way of representing concepts and relationships. Many of these attempts have started with the assumption that students find it easier to master algebra if it is made concrete through the use of manipulables, an assumption supported by many experimental studies (see the meta-analysis of this research by Sowell, 1989). The symbols of algebra are typically reified either in physical objects or in computer representations or simulations of problems and problem-solving (Austin & Vollrath, 1989; Watkins & Taylor, 1989; Waits & Demana, 1989; Shumway, 1989). Usually, students can learn rudimentary aspects of algebra from these techniques.

Of course, the need for students to develop more generic, abstract and powerful conceptual models has not been ignored in algebra instruction. For example, Connell & Ravlin (1988) have proposed a microcomputer-based instructional model for teaching linear equations that uses four types of problems to encourage just this kind of development. Students begin by using manipulables. Then they learn to represent the manipulables in sketches. The next step is to internalize the sketches as mental images. Only then is it possible, through further abstraction, to arrive at a truly algebraic conceptual model.

We have combined the findings of this research with theory of knowledge construction to create a new way of representing Algebra. Spatial Algebra is concrete, intuitive to work with, comprehensive and appealing. It is designed both to make algebra easier to learn and to permit easy implementation of the concepts and operations of algebra in VR.

Spatial Algebra

Our understanding of a concept is tightly connected to the way we represent that concept. Traditionally, mathematics is presented textually. As a consequence, novice errors in elementary algebra, for example, are due as much to misunderstandings of the nature of tokens as they are to miscomprehensions of the mathematical ideas represented by the tokens. This section outlines a spatial algebra by mapping the structure of commutative groups onto the structure of space. We interact with spatial representations through natural behavior in an inclusive environment. When the environment enforces the transformational invariants of algebra, the spatial representation affords experiential learning. Experiential algebra permits algebraic proof through direct manipulation and can be readily implemented in virtual reality. The techniques used to create spatial algebra lay a foundation for the exploration of experiential learning of mathematics in virtual environments.

How we think about mathematical concepts is often constrained by our representation of those concepts. Syntax and semantics (representation and meaning) are tightly connected. The addition operation, for example, is conceptualized as binary when written in linear text:

$$x + y$$

To add three numbers, we must use two addition operations:

$$x + y + z$$

Column addition, however, reconceptualizes the addition operation to be *variary* (one operator can be applied to an arbitrary number of arguments):

```
   x
   y
 + z
-----
```

Naturally, the addition algorithms and techniques taught to students differ for the different representations.

In general, how we represent numbers is a matter of convenience. For learning mathematics (and for doing mathematics) it is often more convenient to call upon visual interaction and natural behavior than it is to conduct symbolic substitutions devoid of meaning. Spatial algebra uses the three dimensions of natural space to express algebraic concepts. A higher dimension of representation greatly simplifies the visualization and the application of algebraic axioms. Algebraic transformation and the process of proof are achieved through direct manipulation of the three-dimensional representation of the algebra problem.

Spatial Representation of Algebra

One possible map from algebraic tokens to algebraic spaces is:

Constants:	{ 1, 2, 3, ...}	-->	{ labeled-blocks }
Variables:	{ x, y, z, ...}	-->	{ labeled-blocks }
Operators:	{ + }	-->	{ sharing-same-space }
	{ * }	-->	{ touching-each-other }
Relations:	{ = }	-->	{ partitions-between-spaces }

Examples of a spatial representation of the above map follow.

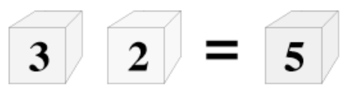
Constant as labeled block:

3

Variable as labeled block:

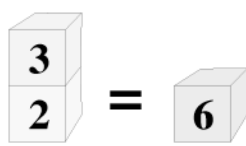
x

Space sharing as *addition*:



3 + 2 = 5

Touching as *multiplication*:



3 * 2 = 6

The gravitational orientation of the typography (top to bottom of page) in the above examples is not an aspect of spatial algebra, although gravitational metaphors are useful for the representation of sequential concepts such as non-commutativity. As well, the sequencing implied by stacked blocks is an artifact of typography; stacks only represent groups of objects touching in space.

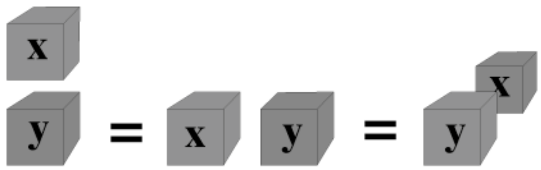
Generally, spatial representation can be mapped onto group theory. A commutative group is a mathematical structure consisting of a set and an operator on elements of that set, with the following properties:

- The set is closed under the operation.
- The operation is associative and commutative.
- There is an identity element.
- Every element has an inverse.

The integer addition and multiplication operators taught in elementary school belong to the commutative group.

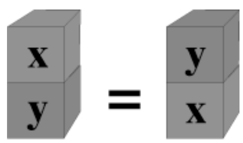
Commutativity

Spatial representation permits the implicit embedding of commutativity in space. The *commutativity of addition* is represented by the absence of linear ordering of blocks in space (visualize the blocks in this example as floating in space rather than in a particular linear order):



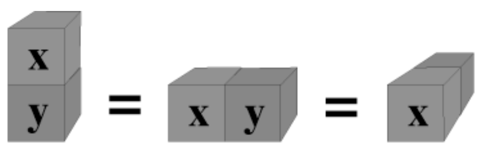
x + y = y + x

Commutativity of multiplication can be seen as the absence of ordering in touching blocks:



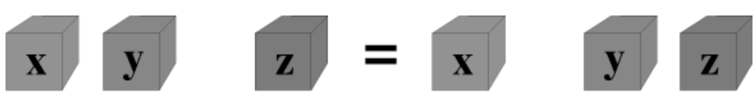
x * y = y * x

Again, in space there is no preferential ordering to touching objects:



Associativity

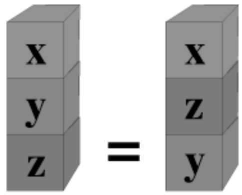
Associativity of addition is the absence of an explicit grouping concept in space:



(x + y) + z = x + (y + z)

From an intuitive perspective, operations embedded in space apply to any number of objects in that space. Whatever grouping we use is a matter a choice and convenience.

Associativity of multiplication is the absence of an explicit grouping concept in piles:

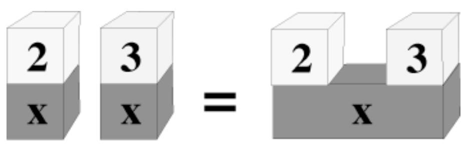


$$(x * y) * z = (x * z) * y$$

The apparent visual ordering of piles can be overcome by assuming that all objects in a pile touch one another directly. Rather than displaying stacked objects, VR might present objects in piles as completely interpenetrating. Every object in this non-physical representation is in contact with every other object, forming a Cartesian product of touching objects.

Distribution

Precedence operations associated with the distributive rule are the most common algebraic error for first year students (National Assessment, 1981; Bricken, 1987). The representation of distribution in spatial algebra is particularly compelling. Example:



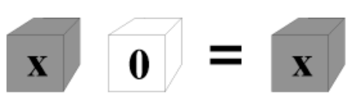
$$2x + 3x = (2 + 3)x$$

Generally, the *distributive law* permits combining blocks with identical labels into a single block with that label. The ability to arbitrarily divide and combine blocks with a common name is the same as the ability to arbitrarily create duplicate labels in a textual representation. Changing the size and the number of occurrences of a labeled block is easy in a virtual environment.

Identities

Zero is the identity element for addition. The identity in the spatial metaphor is the void; identities are equivalent to empty space.

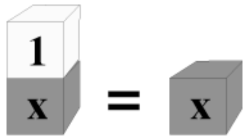
The *additive identity*:



$$x + 0 = x$$

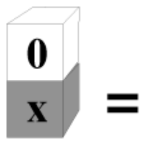
That is, zero disappears in space:

The *multiplicative identity*:



1 * x = x

The One block disappears only in the context of an existing pile. A zero in a pile makes the entire pile disappear:

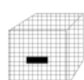


0 * x = 0

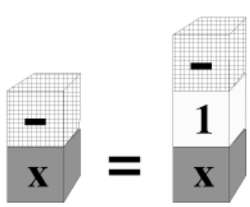
Additive Inverse

The inverse of a positive number is a negative number. Negative numbers are the most difficult aspect of arithmetic for elementary students. One way to directly represent inversion is to create an inverter block. Another way is to create an inversion space; for example using "under-the-table" for inverses. Inverses can be represented in many ways: as inverters, as colors, as orientations, as different spaces, as binary switches, as dividing planes, as inside-out objects.

In this version of spatial algebra, piles are inverted by the inclusion of an inverter block:



Since a negative number can be seen as being multiplied by -1, the inverter block is expressed as touching (multiplying) the pile which is inverted:

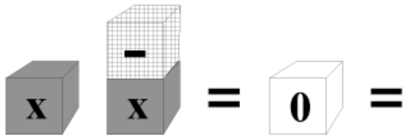


-x = (-1) * x

The inverter block expresses subtraction as the addition of inverses:

x - x = x + (-x)

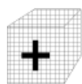
The *additive inverse*:



x + (-x) = 0

Calculus of Signs

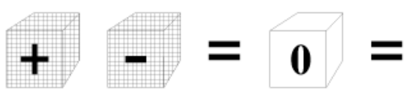
The use of the inverter block for negative numbers introduces a calculus of signs into the algebra of integers. A sign calculus requires the explicit introduction of the positive block:



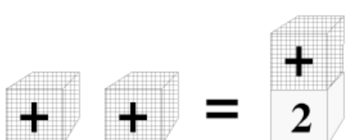
The positive block is the inverse of the inverter block. It introduces the concept of polarity and the act of cancellation. Numbers without signs are usually assumed to be positive. Making signs explicit removes this assumption.

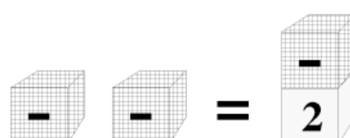
The following rules of sign calculus assume each sign has a unit value associated with it.

Additive cancellation in space:

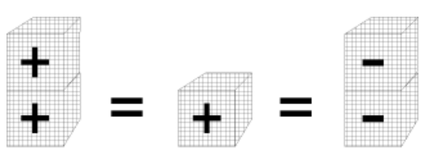


Cardinality in space:

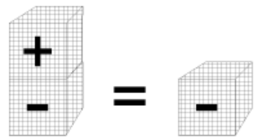




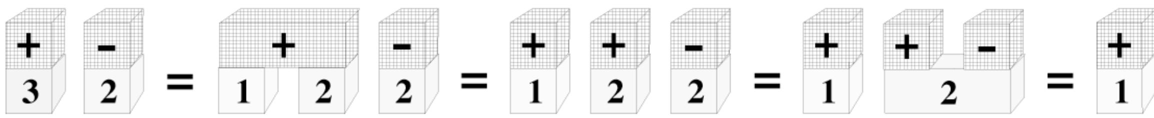
Multiplicative cancellation in piles:



Multiplicative dominance in piles:



The calculus of signs permits a model of arithmetic simplification. This example of integer subtraction graphically illustrates the distribution and cancellation processes: 3 - 2 = 1

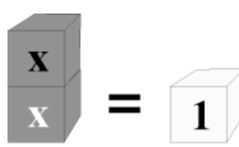


Multiplicative Inverse

Finally, division is the multiplicative inverse. Again, there are many possible ways to represent an inverse in a spatial representation. Since the traditional notation for fractions is primarily two-dimensional, it already has many spatial aspects. The division line that separates numerator from denominator could be carried over to the spatial representation as a plane dividing a pile into two parts. Here however, the multiplicative inverse is represented by inverse shading of the block label:

1/x

The *multiplicative inverse*:

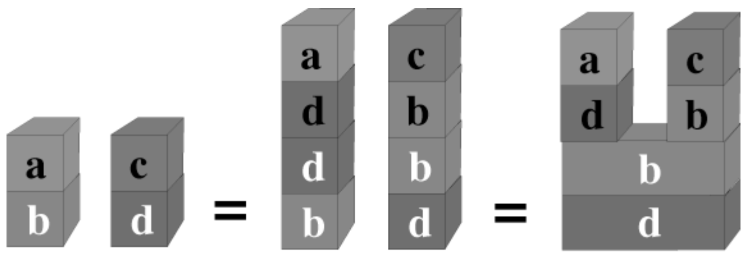


x * 1/x = 1

In the sign calculus, a sign is its own multiplicative inverse:

One weakness with the choice to represent a reciprocal as differently shaded labels is that composition of reciprocals -- for example: 1/(1/x) -- is not visually defined. Choice of representation necessarily effects pedagogy. It is an empirical question as to which representations facilitate learning algebraic concepts efficiently.

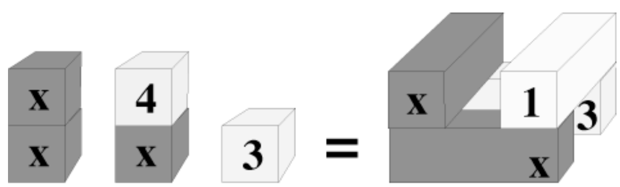
Fractions are the second most difficult area for students of arithmetic. A typical problem using fractions requires the application of the distributive rule:



$a/b + c/d = (ad + bc)/bd$

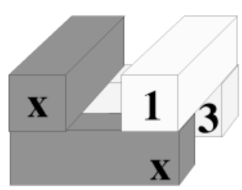
Factoring

Factoring polynomial expressions is equivalent to multiple applications of distribution. For example:



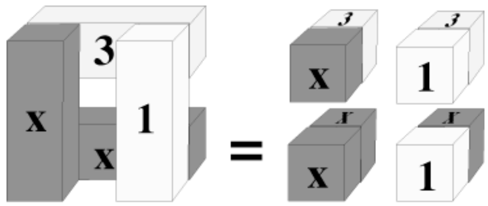
$x^2 + 4x + 3 = (x + 1)*(x + 3)$

One advantage of the spatial representation on the right-hand-side of this equation is that both the factored and the polynomial forms are visible concurrently. Looking from the side, we see two completely touching spaces which represent the factored form:



$(x + 1) * (x + 3)$

Looking down from the top, we see four piles which represent the polynomial form:



x^2 + 1*x + 3*x + 1*3

Here, the factored form is converted to the polynomial by slicing each addition space through the middle.

Conclusion: Why Use VR in School?

On the basis of the research and theory that we have discussed above, we have concluded that VR has the potential for making a significant improvement in the way students learn Mathematics. Our approach has been to identify a clear application of VR techniques to evaluate in a classroom application. We believe that Spatial Algebra allows the concepts and procedures of algebra to be represented in a virtual world that embodies constructivist learning principles, while at the same time supporting the kind of pedagogical strategies that are required when students build knowledge in structured domains. More generally, the following observations contribute to our confidence that VR has a great deal to contribute to providing effective learning experiences to students.

Virtual worlds are totally engaging, entirely immersing the student cognitively and affectively in the environment. VR places the participant in a three-dimensional visual, auditory and visceral environment. The sensations that the participant receives are pervasive and convincing. Evidence from a study of children working with VR (Bricken & Byrne, 1992) has shown that it is indeed engaging and motivating.

Interaction with the virtual world is intuitive because students interact with objects in natural ways, by grasping, pointing, etc. The interface is often an impediment to interaction with traditional CAI systems. At the very least, the keyboard, mouse, screen or touch pad come between the student and the program. In VR, there is no interface (M. Bricken, 1991a). The participant interacts directly with the objects in the virtual world. In a very real sense, the participant's natural actions and the participant's senses replace the interface. Because the participant's actions are mapped by the program in virtual space, the participant is part of the program.

The virtual world can be programmed to provide various types of guidance to students. Virtual worlds, like simulations, are programmed environments with which participants can interact in real time. VR goes further than simulations, however, in that virtual worlds can embody arbitrary objects, abstract or concrete, and can be programmed to behave in ways that have no equivalence in the real world. The great strength of the algebra world is its programmability, which allows it to embody the rules of Algebra.

Virtual objects behave in concrete ways. In addition to its embodiment of the rules of algebra, the algebra world provides students with the demonstrated advantages of manipulable concrete objects which, as we saw earlier, help students construct knowledge of algebra. The student can manipulate virtual objects in the same way that real objects are manipulated -- by picking them up, moving them, turning them over, combining them, and so on.

Students can explore and return to the same place repeatedly, building an increasingly sophisticated understanding of concepts and procedures. A virtual world is created by a non-linear program. (It is "object-oriented" in the computer science sense too.) This means that participants act freely in a responsive environment. They can return many times to the same object and move freely in the virtual world for activity to activity, and even from time to time. This ability to "criss-cross the landscape" (Spiro et al., 1991) is, as we have seen, a necessary strategy for knowledge construction by students.

The system can automate some procedures, allowing students to concentrate on others. Finally, should the need arise, the algebra world can take over some tasks from the student, allowing the student to concentrate on those tasks that need more practice. For example, the arithmetic involved in factoring an expression can be automated by the system so that it ceases to distract the student from other operations.

For these reasons, we are convinced that VR has great educational potential. As research and development in the Human Interface Technology Laboratory and the College of Education proceeds, we are hopeful that the scope and constraints of VR's potential will become clear. We do not expect that VR will revolutionize education. Nor do we expect that worlds and the systems to run them will be generally available for a number of years. However, we are encouraged by the speed at which research and development are proceeding, and look forward to assessing the effectiveness of VR in math education and in other areas of the curriculum.

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