PRINCIPLES FOR SCHOOL MATHEMATICS

Decisions made by teachers, school administrators, and other education professionals about the content and character of school mathematics have important consequences both for students and for society. These decisions should be based on sound professional guidance. Principles and Standards for School Mathematics is intended to provide such guidance. The Principles describe particular features of high-quality mathematics education. The Standards describe the mathematical content and processes that students should learn. Together, the Principles and Standards constitute a vision to guide educators as they strive for the continual improvement of mathematics education in classrooms, schools, and educational systems.

The six principles for school mathematics address overarching themes:

- ***Equity***. Excellence in mathematics education requires equity—high expectations and strong support for all students.

- ***Curriculum***. A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

- ***Teaching***. Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

- ***Learning***. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

- ***Assessment***. Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

- ***Technology***. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

These six Principles, which are discussed in depth below, do not refer to specific mathematics content or processes and thus are quite different from the Standards. They describe crucial issues that, although not unique to school mathematics, are deeply intertwined with school mathematics programs. They can influence the development of curriculum frameworks, the selection of curriculum materials, the planning of instructional units or lessons, the design of assessments, the assignment of teachers and students to classes, instructional decisions in the classroom, and the establishment of supportive professional development programs for teachers. The perspectives and assumptions underlying the Principles are compatible with, and foundational to, the Standards and expectations presented in chapters 3–7.

Each Principle is discussed separately, but the power of these Principles as guides and tools for decision making derives from their interaction in the thinking of educators. The Principles will come fully alive as they are used together to develop high-quality school mathematics programs.

The Equity Principle

Excellence in mathematics education requires equity—high expectations and strong support for all students.

Making the vision of the Principles and Standards for School Mathematics a reality for all students, prekindergarten through grade 12, is both an essential goal and a significant challenge. Achieving this goal requires raising expectations for students' learning, developing effective methods of supporting the learning of mathematics by all students, and providing students and teachers with the resources they need.

Educational equity is a core element of this vision. All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students.

Equity is interwoven with the other Principles. All students need access each year to a coherent, challenging mathematics curriculum taught by competent and well-supported mathematics teachers. Moreover, students' learning and achievement should be assessed and reported in ways that point to areas requiring prompt additional attention. Technology can assist in achieving equity and must be accessible to all students.

Equity requires high expectations and worthwhile opportunities for all.

The vision of equity in mathematics education challenges a pervasive societal belief in North America that only some students are capable of learning mathematics. This belief, in contrast to the equally pervasive view that all students can and should learn to read and write in English, » leads to low expectations for too many students. Low expectations are especially problematic because students who live in poverty, students who are not native speakers of English, students with disabilities, females, and many nonwhite students have traditionally been far more likely than their counterparts in other demographic groups to be the victims of low expectations. Expectations must be raised—mathematics can and must be learned by all students.

The Equity Principle demands that high expectations for mathematics learning be communicated in words and deeds to all students. Teachers communicate expectations in their interactions with students during classroom instruction, through their comments on students' papers, when assigning students to instructional groups, through the presence or absence of consistent support for students who are striving for high levels of attainment, and in their contacts with significant adults in a student's life. These actions, along with decisions and actions taken outside the classroom to assign students to different classes or curricula, also determine students' opportunities to learn and influence students' beliefs about their own abilities to succeed in mathematics. Schools have an obligation to ensure that all students participate in a strong instructional program that supports their mathematics learning. High expectations can be achieved in part with instructional programs that are interesting for students and help them see the importance and utility of continued mathematical study for their own futures.

Equity requires accommodating differences to help everyone learn mathematics.

Higher expectations are necessary, but they are not sufficient to accomplish the goal of an equitable school mathematics education for all students. All students should have access to an excellent and equitable mathematics program that provides solid support for their learning and is responsive to their prior knowledge, intellectual strengths, and personal interests.

Some students may need further assistance to meet high mathematics expectations. Students who are not native speakers of English, for instance, may need special attention to allow them to participate fully in classroom discussions. Some of these students may also need assessment accommodations. If their understanding is assessed only in English, their mathematical proficiency may not be accurately evaluated.

Students with disabilities may need increased time to complete assignments, or they may benefit from the use of oral rather than written assessments. Students who have difficulty in mathematics may need additional resources, such as after-school programs, peer mentoring, or cross-age tutoring. Likewise, students with special interests or exceptional talent in mathematics may need enrichment programs or additional resources to challenge and engage them. The talent and interest of these students must be nurtured and supported so that they have the opportunity and guidance to excel. Schools and school systems must take care to accommodate the special needs of some students without inhibiting the learning of others.

Technology can help achieve equity in the classroom. For example, technological tools and environments can give all students opportunities » to explore complex problems and mathematical ideas, can furnish structured tutorials to students needing additional instruction and practice on skills, or can link students in rural communities to instructional opportunities or intellectual resources not readily available in their locales. Computers with voice-recognition or voice-creation software can offer teachers and peers access to the mathematical ideas and arguments developed by students with disabilities who would otherwise be unable to share their thinking. Moreover, technology can be effective in attracting students who disengage from nontechnological approaches to mathematics. It is important that all students have opportunities to use technology in appropriate ways so that they have access to interesting and important mathematical ideas. Access to technology must not become yet another dimension of educational inequity.

Equity requires resources and support for all classrooms and all students.

Well-documented examples demonstrate that all children, including those who have been traditionally underserved, can learn mathematics when they have access to high-quality instructional programs that support their learning (Campbell 1995; Griffin, Case, and Siegler 1994; Knapp et al. 1995; Silver and Stein 1996). These examples should become the norm rather than the exception in school mathematics education.

Achieving equity requires a significant allocation of human and material resources in schools and classrooms. Instructional tools, curriculum materials, special supplemental programs, and the skillful use of community resources undoubtedly play important roles. An even more important component is the professional development of teachers. Teachers need help to understand the strengths and needs of students who come from diverse linguistic and cultural backgrounds, who have specific disabilities, or who possess a special talent and interest in mathematics. To accommodate differences among students effectively and sensitively, teachers also need to understand and confront their own beliefs and biases.

The Curriculum Principle

A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

A school mathematics curriculum is a strong determinant of what students have an opportunity to learn and what they do learn. In a coherent curriculum, mathematical ideas are linked to and build on one another so that students' understanding and knowledge deepens and their ability to apply mathematics expands. An effective mathematics curriculum focuses on important mathematics—mathematics that will » prepare students for continued study and for solving problems in a variety of school, home, and work settings. A well-articulated curriculum challenges students to learn increasingly more sophisticated mathematical ideas as they continue their studies.

A mathematics curriculum should be coherent.

Mathematics comprises different topical strands, such as algebra and geometry, but the strands are highly interconnected. The interconnections should be displayed prominently in the curriculum and in instructional materials and lessons. A coherent curriculum effectively organizes and integrates important mathematical ideas so that students can see how the ideas build on, or connect with, other ideas, thus enabling them to develop new understandings and skills.

Curricular coherence is also important at the classroom level. Researchers have analyzed lessons in the videotape study of eighth-grade mathematics classrooms that was part of the Third International Mathematics and Science Study (Stigler and Hiebert 1999). One important characteristic of the lessons had to do with the internal coherence of the mathematics. The researchers found that typical Japanese lessons were designed around one central idea, which was carefully developed and extended; in contrast, typical American lessons included several ideas or topics that were not closely related and not well developed.

In planning individual lessons, teachers should strive to organize the mathematics so that fundamental ideas form an integrated whole. Big ideas encountered in a variety of contexts should be established carefully, with important elements such as terminology, definitions, notation, concepts, and skills emerging in the process. Sequencing lessons coherently across units and school years is challenging. And teachers also need to be able to adjust and take advantage of opportunities to move lessons in unanticipated directions.

A mathematics curriculum should focus on important mathematics.

School mathematics curricula should focus on mathematics content and processes that are worth the time and attention of students. Mathematics topics can be considered important for different reasons, such as their utility in developing other mathematical ideas, in linking different areas of mathematics, or in deepening students' appreciation of mathematics as a discipline and as a human creation. Ideas may also merit curricular focus because they are useful in representing and solving problems within or outside mathematics.

Foundational ideas like place value, equivalence, proportionality, function, and rate of change should have a prominent place in the mathematics curriculum because they enable students to understand other mathematical ideas and connect ideas across different areas of mathematics. Mathematical thinking and reasoning skills, including making conjectures and developing sound deductive arguments, are important because they serve as a basis for developing new insights and promoting further study. Many concepts and processes, such as symmetry and generalization, can help students gain insights into the nature and beauty of

mathematics. In addition, the curriculum should offer experiences that » allow students to see that mathematics has powerful uses in modeling and predicting real-world phenomena. The curriculum also should emphasize the mathematics processes and skills that support the quantitative literacy of students. Members of an intelligent citizenry should be able to judge claims, find fallacies, evaluate risks, and weigh evidence (Price 1997).

Although any curriculum document is fixed at a point in time, the curriculum itself need not be fixed. Different configurations of important mathematical ideas are possible and to some extent inevitable. The relative importance of particular mathematics topics is likely to change over time in response to changing perceptions of their utility and to new demands and possibilities. For example, mathematics topics such as recursion, iteration, and the comparison of algorithms are receiving more attention in school mathematics because of their increasing relevance and utility in a technological world.

A mathematics curriculum should be well articulated across the grades.

Learning mathematics involves accumulating ideas and building successively deeper and more refined understanding. A school mathematics curriculum should provide a road map that helps teachers guide students to increasing levels of sophistication and depths of knowledge. Such guidance requires a well-articulated curriculum so that teachers at each level understand the mathematics that has been studied by students at the previous level and what is to be the focus at successive levels. For example, in grades K–2 students typically explore similarities and differences among two-dimensional shapes. In grades 3–5 they can identify characteristics of various quadrilaterals. In grades 6–8 they may examine and make generalizations about properties of particular quadrilaterals. In grades 9–12 they may develop logical arguments to justify conjectures about particular polygons. As they reach higher levels, students should engage more deeply with mathematical ideas and their understanding and ability to use the knowledge is expected to grow.

Without a clear articulation of the curriculum across all grades, duplication of effort and unnecessary review are inevitable. A well-articulated curriculum gives teachers guidance regarding important ideas or major themes, which receive special attention at different points in time. It also gives guidance about the depth of study warranted at particular times and when clos

The Teaching Principle

Effective mathematics teaching requires understanding what students know and need to learn ***and then challenging and supporting them to learn it well.***

Students learn mathematics through the experiences that teachers provide. Thus, students' understanding of mathematics, their ability to » use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school. The improvement of mathematics education for all students requires effective mathematics teaching in all classrooms.

Teaching mathematics well is a complex endeavor, and there are no easy recipes for helping all students learn or for helping all teachers become effective. Nevertheless, much is known about effective mathematics teaching, and this knowledge should guide professional judgment and activity. To be effective, teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks. They need to understand and be committed to their students as learners of mathematics and as human beings and be skillful in choosing from and using a variety of pedagogical and assessment strategies (National Commission on Teaching and America's Future 1996). In addition, effective teaching requires reflection and continual efforts to seek improvement. Teachers must have frequent and ample opportunities and resources to enhance and refresh their knowledge.

The Professional Standards for Teaching Mathematics (NCTM, 1991) presented six standards for the teaching of mathematics. They addressed–

* worthwhile mathematical tasks;
* the teacher's role in discourse;
* the student's role in discourse;
* tools for enhancing discourse;
* the learning environment;
* the analysis of teaching and learning.

Effective teaching requires knowing and understanding mathematics, students as learners, and ***pedagogical strategies.***

Teachers need several different kinds of mathematical knowledge—knowledge about the whole domain; deep, flexible knowledge about curriculum goals and about the important ideas that are central to their grade level; knowledge about the challenges students are likely to encounter in learning these ideas; knowledge about how the ideas can be represented to teach them effectively; and knowledge about how students' understanding can be assessed. This knowledge helps teachers make curricular judgments, respond to students' questions, and look ahead to where concepts are leading and plan accordingly. Pedagogical knowledge, much of which is acquired and shaped through the practice of teaching, helps teachers understand how students learn mathematics, become facile with a range of different teaching techniques and instructional materials, and organize and manage the classroom. Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise (Schifter 1999; Ma 1999). Their decisions and their actions in the classroom—all of which affect how well their students learn mathematics—should be based on this knowledge.

This kind of knowledge is beyond what most teachers experience in standard preservice mathematics courses in the United States. For example, that fractions can be understood as parts of a whole, the quotient of two

integers, or a number on a line is important for mathematics teachers (Ball and Bass forthcoming). Such understanding might be characterized as "profound understanding of fundamental mathematics" (Ma 1999). Teachers also need to understand the different representations of an idea, the relative strengths and weaknesses of each, and how they are related to one another (Wilson, Shulman, and Richert 1987). They need to know the ideas with which students often have difficulty and ways to help bridge common misunderstandings. »

Effective mathematics teaching requires a serious commitment to the development of students' understanding of mathematics. Because students learn by connecting new ideas to prior knowledge, teachers must understand what their students already know. Effective teachers know how to ask questions and plan lessons that reveal students' prior knowledge; they can then design experiences and lessons that respond to, and build on, this knowledge.

Teachers have different styles and strategies for helping students learn particular mathematical ideas, and there is no one "right way" to teach. However, effective teachers recognize that the decisions they make shape students' mathematical dispositions and can create rich settings for learning. Selecting and using suitable curricular materials, using appropriate instructional tools and techniques, and engaging in reflective practice and continuous self-improvement are actions good teachers take every day.

One of the complexities of mathematics teaching is that it must balance purposeful, planned classroom lessons with the ongoing decision making that inevitably occurs as teachers and students encounter unanticipated discoveries or difficulties that lead them into uncharted territory. Teaching mathematics well involves creating, enriching, maintaining, and adapting instruction to move toward mathematical goals, capture and sustain interest, and engage students in building mathematical understanding.

Effective teaching requires a challenging and supportive classroom learning environment.

Teachers make many choices each day about how the learning environment will be structured and what mathematics will be emphasized. These decisions determine, to a large extent, what students learn. Effective teaching conveys a belief that each student can and is expected to understand mathematics and that each will be supported in his or her efforts to accomplish this goal.

Teachers establish and nurture an environment conducive to learning mathematics through the decisions they make, the conversations they orchestrate, and the physical setting they create. Teachers' actions are what encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions. The teacher is responsible for creating an intellectual environment where serious mathematical thinking is the norm. More than just a physical setting with desks, bulletin boards, and posters, the classroom environment communicates subtle messages about what is valued in learning and doing mathematics. Are students' discussion and collaboration encouraged? Are students expected to justify their thinking? If students are to learn to make conjectures, experiment with various approaches to solving problems, construct mathematical arguments and respond to others' arguments, then creating an environment that fosters these kinds of activities is essential.

In effective teaching, worthwhile mathematical tasks are used to introduce important mathematical ideas and to engage and challenge students intellectually. Well-chosen tasks can pique students' curiosity and draw them into mathematics. The tasks may be connected to the » real-world experiences of students, or they may arise in contexts that are purely mathematical. Regardless of the context, worthwhile tasks should be intriguing, with a level of challenge that invites speculation and hard work. Such tasks often can be approached in more than one way, such as using an arithmetic counting approach, drawing a geometric diagram and enumerating possibilities, or using algebraic equations, which makes the tasks accessible to students with varied prior knowledge and experience.

Worthwhile tasks alone are not sufficient for effective teaching. Teachers must also decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge.

Effective teaching requires continually seeking improvement.

Effective teaching involves observing students, listening carefully to their ideas and explanations, having mathematical goals, and using the information to make instructional decisions. Teachers who employ such practices motivate students to engage in mathematical thinking and reasoning and provide learning opportunities that challenge students at all levels of understanding. Effective teaching requires continuing efforts to learn and improve. These efforts include learning about mathematics and pedagogy, benefiting from interactions with students and colleagues, and engaging in ongoing professional development and self-reflection.
Opportunities to reflect on and refine instructional practice—during class and outside class, alone and with others—are crucial in the vision of school mathematics outlined in Principles and Standards. To improve their mathematics instruction, teachers must be able to analyze what they and their students are doing and consider how those actions are affecting students' learning. Using a variety of strategies, teachers should monitor students' capacity and inclination to analyze situations, frame and solve problems, and make sense of mathematical concepts and procedures. They can use this information to assess their students' progress and to appraise how well the mathematical tasks, student discourse, and classroom environment are interacting to foster students' learning. They then use these appraisals to adapt their instruction.

Reflection and analysis are often individual activities, but they can be greatly enhanced by teaming with an experienced and respected colleague, a new teacher, or a community of teachers. Collaborating with colleagues regularly to observe, analyze, and discuss teaching and students' thinking or to do "lesson study" is a powerful, yet neglected, form of professional development in American schools (Stigler and Hiebert 1999). The work and time of teachers must be structured to allow and support professional development that will benefit them and their students. » ure is expected for particular skills or concepts

The Learning Principle

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

The vision of school mathematics in Principles and Standards is based on students' learning mathematics with understanding. Unfortunately, learning mathematics without understanding has long been a common outcome of school mathematics instruction. In fact, learning without understanding has been a persistent problem since at least the 1930s, and it has been the subject of much discussion and research by psychologists and educators over the years (e.g., Brownell [1947]; Skemp [1976]; Hiebert and Carpenter [1992]). Learning the mathematics outlined in chapters 3–7 requires understanding and being able to apply procedures, concepts, and processes. In the twenty-first century, all students should be expected to understand and be able to apply mathematics.

Learning mathematics with understanding is essential.

In recent decades, psychological and educational research on the learning of complex subjects such as mathematics has solidly established the important role of conceptual understanding in the knowledge and activity of persons who are proficient. Being proficient in a complex domain such as mathematics entails the ability to use knowledge flexibly, applying what is learned in one setting appropriately in another. One of the most robust findings of research is that conceptual understanding is an important component of proficiency, along with factual knowledge and procedural facility (Bransford, Brown, and Cocking 1999).

The alliance of factual knowledge, procedural proficiency, and conceptual understanding makes all three components usable in powerful ways. Students who memorize facts or procedures without understanding often are not sure when or how to use what they know, and such learning is often quite fragile (Bransford, Brown, and Cocking 1999). Learning with understanding also makes subsequent learning easier. Mathematics makes more sense and is easier to remember and to apply when students connect new knowledge to existing knowledge in meaningful ways (Schoenfeld 1988). Well-connected, conceptually grounded ideas are more readily accessed for use in new situations (Skemp 1976).

The requirements for the workplace and for civic participation in the contemporary world include flexibility in reasoning about and using quantitative information. Conceptual understanding is an essential component of the knowledge needed to deal with novel problems and settings. Moreover, as judgments change about the facts or procedures that are essential in an increasingly technological world, conceptual understanding becomes even more important. For example, most of the arithmetic and algebraic procedures long viewed as the heart of the school mathematics curriculum can now be performed with handheld calculators. Thus, more attention can be given to understanding the number concepts and the modeling procedures used in solving problems. Change is a ubiquitous feature of » contemporary life, so learning with understanding is essential to enable students to use what they learn to solve the new kinds of problems they will inevitably face in the future.

A major goal of school mathematics programs is to create autonomous learners, and learning with understanding supports this goal. Students learn more and learn better when they can take control of their learning by defining their goals and monitoring their progress. When challenged with appropriately chosen tasks, students become confident in their ability to tackle difficult problems, eager to figure things out on their own, flexible in exploring mathematical ideas and trying alternative solution paths, and willing to

persevere. Effective learners recognize the importance of reflecting on their thinking and learning from their mistakes. Students should view the difficulty of complex mathematical investigations as a worthwhile challenge rather than as an excuse to give up. Even when a mathematical task is difficult, it can be engaging and rewarding. When students work hard to solve a difficult problem or to understand a complex idea, they experience a very special feeling of accomplishment, which in turn leads to a willingness to continue and extend their engagement with mathematics.

Students can learn mathematics with understanding.

Students will be served well by school mathematics programs that enhance their natural desire to understand what they are asked to learn. From a young age, children are interested in mathematical ideas. Through their experiences in everyday life, they gradually develop a rather complex set of informal ideas about numbers, patterns, shapes, quantities, data, and size, and many of these ideas are correct and robust. Thus children learn many mathematical ideas quite naturally even before they enter school (Gelman and Gallistel 1978; Resnick 1987). A pattern of building new learning on prior learning and experience is established early and repeated, albeit often in less obvious ways, throughout the school years (see, e.g., Steffe [1994]). Students of all ages have a considerable knowledge base on which to build, including ideas developed in prior school instruction and those acquired through everyday experience (Bransford, Brown, and Cocking 1999).

The kinds of experiences teachers provide clearly play a major role in determining the extent and quality of students' learning. Students' understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge. Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills (Hanna and Yackel forthcoming). Classroom discourse and social interaction can be used to promote the recognition of connections among ideas and the reorganization of knowledge (Lampert 1986). By having students talk about their informal strategies, teachers can help them become aware of, and build on, their implicit informal knowledge (Lampert 1989; Mack 1990). Moreover, in such settings, procedural fluency and conceptual understanding can be developed through problem solving, reasoning, and argumentation.

The Assessment Principle

Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

When assessment is an integral part of mathematics instruction, it contributes significantly to all students' mathematics learning. When assessment is discussed in connection with standards, the focus is sometimes on using tests to certify students' attainment, but there are other important purposes of assessment. Assessment should be more than merely a test at the end of instruction to see how students perform under special conditions; rather, it should be an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done to students; rather, it should also be done for students, to guide and enhance their learning.

The Assessment Standards for School Mathematics (NCTM, 1995) presented six standards about exemplary mathematics assessment. They addressed how assessment should–

* reflect the mathematics that students should know and be able to do;
* enhance mathematics learning;
* promote equity;
* be an open process;
* promote valid inference;
* be a coherent process.

Assessment should enhance students' learning.

The assertion that assessment should enhance students' learning may be surprising. After all, if assessment ascertains what students have learned and are able to do, how can it also have positive consequences for learning? Research indicates that making assessment an integral part of classroom practice is associated with improved student learning. Black and Wiliam (1998) reviewed about 250 research studies and concluded that the learning of students, including low achievers, is generally enhanced in classrooms where teachers include attention to formative assessment in making judgments about teaching and learning.

Good assessment can enhance students' learning in several ways. First, the tasks used in an assessment can convey a message to students about what kinds of mathematical knowledge and performance are valued. That message can in turn influence the decisions students make—for example, whether or where to apply effort in studying. Thus, it is important that assessment tasks be worthy of students' time and attention. Activities that are consistent with (and sometimes the same as) the activities used in instruction should be included. When teachers use assessment techniques such as observations, conversations and interviews with students, or interactive journals, students are likely to learn through the process of articulating their ideas and answering the teacher's questions.

Feedback from assessment tasks can also help students in setting goals, assuming responsibility for their own learning, and becoming more independent learners. For example, scoring guides, or rubrics, can help teachers analyze and describe students' responses to complex tasks and determine students' levels of proficiency. They can also help students understand the characteristics of a complete and correct response. Similarly, classroom discussions in which students present and evaluate different approaches to solving complex problems can hone their sense of the difference between an excellent response and one that is mediocre. Through the use of good tasks and the public discussion of » criteria for good responses, teachers can cultivate in their students both

the disposition and the capacity to engage in self-assessment and reflection on their own work and on ideas put forth by others. Such a focus on self-assessment and peer assessment has been found to have a positive impact on students' learning (Wilson and Kenney forthcoming).

Assessment is a valuable tool for making instructional decisions.

To ensure deep, high-quality learning for all students, assessment and instruction must be integrated so that assessment becomes a routine part of the ongoing classroom activity rather than an interruption. Such assessment also provides the information teachers need to make appropriate instructional decisions. In addition to formal assessments, such as tests and quizzes, teachers should be continually gathering information about their students' progress through informal means, such as asking questions during the course of a lesson, conducting interviews with individual students, and giving writing prompts.

When teachers have useful information about what students are learning, they can support their students' progress toward significant mathematical goals. The instructional decisions made by teachers—such as how and when to review prerequisite material, how to revisit a difficult concept, or how to adapt tasks for students who are struggling or for those who need enrichment—are based on inferences about what students know and what they need to learn. Assessment is a primary source of the evidence on which these inferences are based, and the decisions that teachers make will be only as good as that evidence.

Assessment should reflect the mathematics that all students need to know and be able to do, and it should focus on students' understanding as well as their procedural skills. Teachers need to have a clear sense of what is to be taught and learned, and assessment should be aligned with their instructional goals. By providing information about students' individual and collective progress toward the goals, assessment can help ensure that everyone moves productively in the right direction.

To make effective decisions, teachers should look for convergence of evidence from different sources. Formal assessments provide only one viewpoint on what students can do in a very particular situation—often working individually on paper-and-pencil tasks, with limited time to complete the tasks. Overreliance on such assessments may give an incomplete and perhaps distorted picture of students' performance. Because different students show what they know and can do in different ways, assessments should allow for multiple approaches, thus giving a well-rounded picture and allowing each student to show his or her best strengths.

Many assessment techniques can be used by mathematics teachers, including open-ended questions, constructed-response tasks, selected-response items, performance tasks, observations, conversations, journals, and portfolios. These methods can all be appropriate for classroom assessment, but some may apply more readily to particular goals. For example, quizzes using simple constructed-response or selected-response items may indicate whether students can apply procedures. » Constructed-response or performance tasks may better illuminate students' capacity to apply mathematics in complex or new situations. Observations and conversations in the classroom can provide insights into students' thinking, and teachers can monitor changes in students' thinking and reasoning over time with reflective journals and portfolios.

When teachers are selecting assessment methods, the age, experience, and special needs of students should be considered. Teachers must ensure that all students have an opportunity to demonstrate clearly and completely what they know and can do. For example, teachers should use English-enhancing and bilingual techniques to support students who are learning English.

When done well, assessment that helps teachers make decisions about the content or form of instruction (often called formative assessment) can also be used to judge students' attainment (summative assessment). The same sources of evidence can be assembled to build a picture of individual students' progress toward the goals of instruction. To maximize the instructional value of assessment, teachers need to move beyond a

superficial "right or wrong" analysis of tasks to a focus on how students are thinking about the tasks. Efforts should be made to identify valuable student insights on which further progress can be based rather than to concentrate solely on errors or misconceptions. Although less straightforward than averaging scores on quizzes, assembling evidence from a variety of sources is more likely to yield an accurate picture of what each student knows and is able to do.

Whether the focus is on formative assessment aimed at guiding instruction or on summative assessment of students' progress, teachers' knowledge is paramount in collecting useful information and drawing valid inferences. Teachers must understand their mathematical goals deeply, they must understand how their students may be thinking about mathematics, they must have a good grasp of possible means of assessing students' knowledge, and they must be skilled in interpreting assessment information from multiple sources. For teachers to attain the necessary knowledge, assessment must become a major focus in teacher preparation and professional development.

The Technology Principle

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

Electronic technologies—calculators and computers—are essential tools for teaching, learning, and doing mathematics. They furnish visual images of mathematical ideas, they facilitate organizing and analyzing data, and they compute efficiently and accurately. They can support investigation by students in every area of mathematics, including geometry, statistics, algebra, measurement, and number. When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving. »

Students can learn more mathematics more deeply with the appropriate use of technology (Dunham and Dick 1994; Sheets 1993; Boers-van Oosterum 1990; Rojano 1996; Groves 1994). Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions. In mathematics-instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics.

The existence, versatility, and power of technology make it possible and necessary to reexamine what mathematics students should learn as well as how they can best learn it. In the mathematics classrooms envisioned in Principles and Standards, every student has access to technology to facilitate his or her mathematics learning under the guidance of a skillful teacher.

Technology enhances mathematics learning.

Technology can help students learn mathematics. For example, with calculators and computers students can examine more examples or representational forms than are feasible by hand, so they can make and explore conjectures easily. The graphic power of technological tools affords access to visual models that are powerful but that many students are unable or unwilling to generate independently. The computational capacity of technological tools extends the range of problems accessible to students and also enables them to execute routine procedures quickly and accurately, thus allowing more time for conceptualizing and modeling.

Students' engagement with, and ownership of, abstract mathematical ideas can be fostered through technology. Technology enriches the range and quality of investigations by providing a means of viewing mathematical ideas from multiple perspectives. Students' learning is assisted by feedback, which technology can supply: drag a node in a Dynamic Geometry® environment, and the shape on the screen changes; change the defining rules for a spreadsheet, and watch as dependent values are modified. Technology also provides a focus as students discuss with one another and with their teacher the objects on the screen and the effects of the various dynamic transformations that technology allows.

Technology offers teachers options for adapting instruction to special student needs. Students who are easily distracted may focus more intently on computer tasks, and those who have organizational difficulties may benefit from the constraints imposed by a computer environment. Students who have trouble with basic procedures can develop and demonstrate other mathematical understandings, which in turn can eventually help them learn the procedures. The possibilities for engaging students with physical challenges in mathematics are dramatically increased with special technologies.

Technology supports effective mathematics teaching.

The effective use of technology in the mathematics classroom depends on the teacher. Technology is not a panacea. As with any teaching tool, it can be used well or poorly. Teachers should use technology to » enhance their students' learning opportunities by selecting or creating mathematical tasks that take advantage of what technology can do efficiently and well—graphing, visualizing, and computing. For example, teachers can use simulations to give students experience with problem situations that are difficult to create without technology, or they can use data and resources from the Internet and the World Wide Web to design student tasks. Spreadsheets, dynamic geometry software, and computer microworlds are also useful tools for posing worthwhile problems.

Technology does not replace the mathematics teacher. When students are using technological tools, they often spend time working in ways that appear somewhat independent of the teacher, but this impression is misleading. The teacher plays several important roles in a technology-rich classroom, making decisions that affect students' learning in important ways. Initially, the teacher must decide if, when, and how technology will be used. As students use calculators or computers in the classroom, the teacher has an opportunity to observe the students and to focus on their thinking. As students work with technology, they may show ways of thinking about mathematics that are otherwise often difficult to observe. Thus, technology aids in assessment, allowing teachers to examine the processes used by students in their mathematical investigations as well as the results, thus enriching the information available for teachers to use in making instructional decisions.

Technology influences what mathematics is taught.

Technology not only influences how mathematics is taught and learned but also affects what is taught and when a topic appears in the curriculum. With technology at hand, young children can explore and solve problems involving large numbers, or they can investigate characteristics of shapes using dynamic geometry software. Elementary school students can organize and analyze large sets of data. Middle-grades students can study linear relationships and the ideas of slope and uniform change with computer representations and by performing physical experiments with calculator-based-laboratory systems. High school students can use simulations to study sample distributions, and they can work with computer algebra systems that efficiently perform most of the symbolic manipulation that was the focus of traditional high school mathematics programs. The study of algebra need not be limited to simple situations in which symbolic manipulation is relatively straightforward. Using technological tools, students can reason about more-general issues, such as parameter changes, and they can model and solve complex problems that were heretofore inaccessible to them. Technology also blurs some of the artificial separations among topics in algebra, geometry, and data analysis by allowing students to use ideas from one area of mathematics to better understand another area of mathematics.

Technology can help teachers connect the development of skills and procedures to the more general development of mathematical understanding. As some skills that were once considered essential are rendered less necessary by technological tools, students can be asked to work at higher levels of generalization or abstraction. Work with virtual manipulatives (computer simulations of physical manipulatives) or with Logo can allow young children to extend physical experience and » to develop an initial understanding of sophisticated ideas like the use of algorithms. Dynamic geometry software can allow experimentation with families of geometric objects, with an explicit focus on geometric transformations. Similarly, graphing utilities facilitate the exploration of characteristics of classes of functions. Because of technology, many topics in discrete mathematics take on new importance in the contemporary mathematics classroom; the boundaries of the mathematical landscape are being transformed.