

## Logical Tautologies

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|-----|--|---------------------------|
| 1.  | $P \vee \sim P$  | excluded middle           |
| 2.  | $\sim(P \ \& \ \sim P)$  | noncontradiction          |
| 3.  | $\sim\sim P = P$   | double negation           |
| 4.  | $(P \ \& \ Q) \rightarrow P$   | simplification            |
| 5.  | $P \rightarrow (P \vee Q)$   | simplification            |
| 6.  | $(P \ \& \ P) = P$   | idempotence               |
| 7.  | $(P \vee P) = P$   | idempotence               |
| 8.  | $(P \ \& \ (P \rightarrow Q)) \rightarrow Q$   | modus ponens              |
| 9.  | $((P \rightarrow Q) \ \& \ (Q \rightarrow R)) \rightarrow (P \rightarrow R)$               | sylogism                  |
| 10. | $(P \rightarrow Q) = (\sim Q \rightarrow \sim P)$  | contraposition            |
| 11. | $((P \rightarrow Q) \ \& \ \sim Q) \rightarrow \sim P$                                     | modus tollens             |
| 12. | $((P \vee Q) \ \& \ \sim P) \rightarrow Q$   | disjunctive syllogism     |
| 13. | $(P \rightarrow Q) = (\sim P \vee Q)$  | conditional disjunction   |
| 14. | $(\sim P \rightarrow (Q \ \& \ \sim Q)) \rightarrow P$                                     | reductio ad absurdum      |
| 15. | $(( (P \rightarrow R) \ \& \ (Q \rightarrow S)) \ \& \ (P \vee Q)) \rightarrow (R \vee S)$ | dilemma                   |
| 16. | $(P \rightarrow (Q \rightarrow R)) = ((P \ \& \ Q) \rightarrow R)$                         | exportation               |
| 17. | $(P = Q) = ((P \rightarrow Q) \ \& \ (Q \rightarrow P))$                                   | biconditional             |
| 18. | $\sim(P \vee Q) = (\sim P \ \& \ \sim Q)$  | DeMorgan                  |
| 19. | $\sim(P \ \& \ Q) = (\sim P \vee \sim Q)$  | DeMorgan                  |
| 20. | $\sim(P \rightarrow Q) = (P \ \& \ \sim Q)$  | negation of conditional   |
| 21. | $\sim(P = Q) = (\sim P = Q)$   | negation of biconditional |

Mathematical Foundations

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|-----|---|-------------------------|
| 22. | $(P \vee Q) = (Q \vee P)$   | commutativity           |
| 23. | $(P \& Q) = (Q \& P)$   | commutativity           |
| 24. | $(P = Q) = (Q = P)$   | commutativity           |
| 25. | $((P \vee Q) \vee R) = (P \vee (Q \vee R))$                               | associativity           |
| 26. | $((P \& Q) \& R) = (P \& (Q \& R))$                                       | associativity           |
| 27. | $((P = Q) = R) = (P = (Q = R))$   | associativity           |
| 28. | $(P \& (Q \vee R)) = ((P \& Q) \vee (P \& R))$                            | distribution            |
| 29. | $(P \vee (Q \& R)) = ((P \vee Q) \& (P \vee R))$                          | distribution            |
| 30. | $(P \rightarrow (Q \vee R)) = ((P \rightarrow Q) \vee (P \rightarrow R))$ | distribution            |
| 31. | $(P \rightarrow (Q \& R)) = ((P \rightarrow Q) \& (P \rightarrow R))$     | distribution            |
| 32. | $((P \vee Q) \rightarrow R) = ((P \rightarrow R) \& (Q \rightarrow R))$   | disjunction/conditional |
| 33. | $((P \& Q) \rightarrow R) = ((P \rightarrow R) \vee (Q \rightarrow R))$   | conjunction/conditional |
| 34. | $(P \rightarrow Q) \rightarrow ((R \& P) \rightarrow (R \& Q))$           | factorization           |
| 35. | $(P \rightarrow Q) \rightarrow ((R \vee P) \rightarrow (R \vee Q))$       | summation               |