Chapter 0   Exercises

Almost all mathematical textbooks begin their story in the middle, as if you already knew the assumptions, critical choices, and relevance of each topic. For example, your text begins to teach about proof theory on page 1.

The following questions come prior to studying the content of mathematics. Answer each question in *one sentence or less*. These questions identify what you believe about mathematics, therefore there are no right or wrong answers, just answers that are less or more thoughtful.

HAND IN YOUR ANSWERS AT THE BEGINNING OF CLASS.

Science

1. Which of the following *physically exist*?
   a. electrons
   b. the temperature of the center of the Sun
   c. the cosmological big bang
   d. the state of the internet

2. Is *length* an objective concept;
      that is, is the length of an object independent of a particular observer?

3. Can you know something about the world without changing the world?

Mathematics

4. Are mathematical ideas invented, discovered, or something else?

5. What is mathematical Truth?

6. In what sense does π exist? Where might it exist?
      Is π a constant? Is it eternal or might it change over time?

7. Are there any mathematical concepts that have only one property
      (i.e., are there pure concepts independent of other concepts)?

8. When is the following equation True?: $7 + 8 = 3$

9. Is every number either even or odd? How do you know?

10. Prove the *Pigeonhole Principle*, that N+1 pigeons cannot fit into N holes without sharing.

11. Imagine two points as close together as possible. Is there another point in between them?
      Are there an infinite number of points in between them?

12. Is the following statement True or False or something else?:
    Somewhere in the decimal expansion of π, there are exactly 34 sevens in a row.

13. What kinds of mathematics can be beautiful?

Computation

14. What is the shortest program which will produce a *random number*?

15. Is a *bit* the simplest computational object?

16. Binary computation uses two states, 0 and 1. Is *unary* computation possible?

17. Name three mathematical concepts which cannot be computed.

Attitude

18. How many of the above questions have you thought about before today?

19. Look at the Chapter 0 quotes which follow. Make three lists of names:
    people that you agree with,
    people that you disagree with, and
    people that you do not understand well enough to have an opinion about.

20. *Essay:* Write less than one page in response to this famous question:

    Why is mathematics so unreasonably effective in describing and predicting reality?

Chapter 0   Quotes

Attitude

"Multiplicity ought not be posited without necessity."
      -- William of Occam (1340)

"The acts of the mind, wherein it exerts its power over simple ideas, are chiefly these three:
  1) Combining several simple ideas into one compound one, and thus all complex ideas are made.
  2) The second is bringing two ideas, whether simple or complex, together, and setting them by one another so as to take a view of them at once, without uniting them into one, by which it gets all its ideas of relations.
  3) The third is separating them from all other ideas that accompany them in their real existence; this is called abstraction, and thus all its general ideas are made."
      -- John Locke (1690)

"The language of all art forms, such as cookery, drawing, programming, research, mathematics, and music, is a set of instructions which, if followed, will lead the reader to the same ecstacies as those experienced by the original artist."
      -- G. Spencer-Brown

Science

"As far as the propositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality."
      -- A. Einstein

"The axiomatization and algebraization of mathematics, after more than fifty years, has led to the illegibility of such a large number of mathematical texts that the threat of complete loss of contact with physics and the natural sciences has been realized."
      -- V. I. Arnold

"I believe there are exactly 15,747,724,136,275,002,577,605,653,961,181,555,468,044,717,914,527,116,709, 366,231,425,076,185,631,031,296 protons in the universe, and the same number of electrons."
      -- Sir Arthur Eddington

Mathematics

"Mathematics has always skirted dangerously close to the shores of metaphysics."
      -- S. G. Shanker

"We now come to the decisive step of mathematical abstraction: we forget about what the symbols stand for. ... There are many operations which [the mathematician] may carry out with these symbols, without ever having to look at the things they stand for."
      -- H. Weyl

"Of all escapes from reality, mathematics is the most successful ever. It is a fantasy that becomes all the more addictive because it works back to improve the same reality we are trying to evade. All other escapes -- sex, drugs, hobbies, whatever -- are ephemeral by comparison."
      -- G.C. Rota

"Mathematics, this product of human activity, 'alienates itself' from the human activity which has been producing it. It becomes a living, growing organism, that *acquires a certain autonomy* from the activity which has produced it; it develops its own autonomous laws of growth, its own dialect."
      -- I. Lakatos

Computation

"Computer science also differs from physics in that it is not actually a science. It does not study natural objects. Neither is it, as you might think, mathematics."
      -- R. Feynman

"There is no need for infinities; there are quite enough finite numbers to serve any purpose."
      -- D. Knuth

"Calculation is a method of getting rid of information in which you are not interested. You throw away what is not relevant."
      -- T. Norretranders