Boundary Number Systems -- Introduction

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History of Integers

Number systems evolve in abstraction, computability and expressability.

Sumerian tokens `IIIIII + IIIIIIIIIII = IIIIIIIIIIIIIIIII`

easy to add, easy to multiply, very hard to read

Roman numerals `VI + XI = XVII`

easy to add, hard to multiply, moderate to read

Arabic decimal numbers `6 + 11 = 17`

moderate to add and to multiply, easy to read

Binary numbers `110 + 1011 = 10001`

easy to add and to multiply, moderate to read

Boundary numbers `((*)*) + (((*))*)* = ((*)*)(((*))*)*`

very easy to add and to multiply, hard to read

Types of Numbers

Name	*Property*	*Relation*	*New operation*
indicative	existential	exists or not	
nominal	categorical	share some property	+ attribute, member
ordinal	ranking	put in order	+ less than
integer	discrete	equal intervals	+ equality
rational	comparative	fraction	+ divide and zero
real	continuous	greater infinity	+ compactness
imaginary	complex	impossibility	+ i, other unit bases

The Numerical/Measurement Hierarchy

Name	*Operation*	*Math*
indicative	( )	boundary
nominal	indicative1 member-of indicative2	set
ordinal	nominal1 less-than nominal2	order
integer	ordinal - next-ordinal = constant	discrete
rational	integer1 / integer2	quotient
real	dense packing between rationals	numerical
imaginary	(real1, real2)	complex

Integers as Sets

	Cardinality:	*Ordinality*:	*Uniqueness*:
0			
1	{ }	{ }	{ }
2	{ } { }	{ { } }	{{ }}
3	{ } { } { }	{ { { } } }	{{ },{{ }}}
4	{ } { } { } { }	{ { { { } } } }	{{ },{{ }},{{ },{{ }}}}
n	..n..	''n''	{1,..,n-1}

Some Exotic Varieties of Numbers

Conway numbers (surreals) provide a single coherent framework for defining all types of numbers, and provide ways to manipulate infinite forms. They arise from the act of partitioning nothing.

Spencer-Brown arithmetic is a boundary representation in which each form is both a numerical object and an operator. Arrangements of a single type of boundary token express single numbers, as well as compound operations on multiple numbers.

Kauffman arithmetic uses a boundary form of place notation to provide a more efficient computational representation while maintaining operations which are both parallel and insensitive to the magnitude of a number.

Bricken graph-numbers are an interpretation of Kauffman arithmetic which is desirable for computation. The algebra is represented as graphs rather than strings using parallel graph reduction software.

The *James* Calculus uses three boundaries to shift the representation of numbers between exponential and logarithmic forms. This mechanism generalizes the concepts of cardinality and inverse operations. A *new* imaginary imparts phase structure on numbers, and permits computation without inverses. This system is discussed in depth as an example of novel mathematical thinking, and provides an astonishing link between *imaginary surreals* and *function inversion*.

Boundary Number Systems

Boundary number systems can be characterized by these features:

- semantic use of the void
- semantic use of spatial juxtaposition
- containers as tokens
- object/process confounding
- implicit commutativity and associativity
- a diversity of standard algebraic operations condensed into a few axioms
- computational effort in form standardization rather than addition or multiplication

The last feature is an historical reversion using efficient computational techniques. Place notation and algebraic operations (introduced in the sixteenth century) shift the computational effort from one-to-one correspondence to abstract transformation of structure based on rules. Boundary numbers make the traditional algebraic operations {+,-,*,/,^,root} trivial to implement; the computational effort is shifted to converting a given form into a canonical representation. However, in contrast to conventional decimal and binary numbers, boundary numbers can be read as a computational result at any time during the canonicalization process.

An advantage of the boundary notation is that it can condense a diversity of standard algebraic operations and transformations into three simple axiomatic rules.