

BOOLEAN AND BROWNIAN ALGEBRA

William Bricken

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Here's the score-card for comparing *Boolean algebra* to boundary math's *Brownian algebra*. The most recent breakthrough shows that the *distributive law* is not fundamental. I know of no one else who has proof for this.

Definition: BOOLEAN ALGEBRA {K, &, V, ¬}

K: a set of objects
&: a binary operation on K
V: a dual binary operation on K
¬: a unary operation on K

with AXIOMS:

Associative: $A \& (B \& C) = (A \& B) \& C$ and for dual V

Commutative: $A \& B = B \& A$ and for dual V

Distributive: $A \& (B V C) = (A V B) \& (B V C)$ and for dual V

Zero element: $A V \emptyset = A$

Unit element: $A \& 1 = A$

Complement: $A \& \neg A = \emptyset$
 $A V \neg A = 1$

It is know that associativity is not an axiom, i.e. it can be deduced. In contrast:

Definition: BROWNIAN ALGEBRA {K, ()}

K : a set of objects
(): a variary operator on K (i.e. everything else)

with AXIOMS:

Unit element: $A () = ()$ which is $A V 1 = 1$

Involution: $((A)) = A$ which is $\neg\neg A = A$

Pervasion: $A (A) = A ()$ which is $A V \neg A = A V 1$

In the Brownian algebra, duals are absorbed into the space of representation, so there are no axioms for $\&$ (or alternatively \vee , choose whichever one).

Commutativity is eliminated by creating a representational space that does not support any kind of connectivity between objects in it. Associativity is also eliminated by a representational space that supports no structure.

So the Brownian idea is that Boolean algebra is built from three axioms, not eight (plus two for associativity). One axiom shows how the concept "1" works, one shows how " \neg " works, and the third shows a generalized meaning for " \vee ".

The gain:

no dual operators (saves 3 axioms)
no dual constant (saves 1)
no order (saves 1)

What remains:

LoF

unit
involute
pervasion

BA

unit
complement
distribution