KAUFFMAN'S SINGLE AXIOM AND ITS VARIETIES

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Kauffman's single axiom is formatted as a tableau (after Kauffman), and manipulated to show symmetries.

Theorem(s) **CANCELLATION**

```
      1            ( A  B)( A (B))  = (A)
      2            ((A) B)((A)(B))  =  A
      3           (( A  B)( A (B))) =  A
      4           (((A) B)((A)(B))) = (A)
```

I'll standardize each equation, using [] to highlight

```
            X = Y  ==>  [X Y] [[X][Y]]          Standardization
```

```
      [ ( A  B)( A (B))  (A)] [ [ ( A  B)( A (B)) ] [(A)] ]
      [ ((A) B)((A)(B))   A ] [ [ ((A) B)((A)(B)) ] [ A ] ]
      [(( A  B)( A (B)))  A ] [ [(( A  B)( A (B)))] [ A ] ]
      [(((A) B)((A)(B))) (A)] [ [(((A) B)((A)(B)))] [(A)] ]
```

Involution:

```
      [ ( A  B)( A (B))  (A)] [ [ ( A  B)( A (B)) ]   A   ]
      [ ((A) B)((A)(B))   A ] [ [ ((A) B)((A)(B)) ] [ A ] ]
      [(( A  B)( A (B)))  A ] [   ( A  B)( A (B))   [ A ] ]
      [(((A) B)((A)(B))) (A)] [   ((A) B)((A)(B))     A   ]
```

Literal Pervasion/subsumption by A:

```
      [                  (A)] [ [ (    B)(   (B)) ]   A   ]
      [ (( ) B)(( )(B))   A ] [ [ (    B)(   (B)) ] [ A ] ]
      [((    B)(   (B)))  A ] [                     [ A ] ]
      [((    B)(   (B))) (A)] [   (( ) B)(( )(B))     A   ]
```

Subsumption directly eliminates the B subforms two cases. All the rest simplify by Pervasion. Next apply Occlusion to eliminate two more B subforms.

```
      [                  (A)] [ [ (    B)(   (B)) ]   A   ]
      [                   A ] [ [ (    B)(   (B)) ] [ A ] ]
      [((    B)(   (B)))  A ] [                     [ A ] ]
      [((    B)(   (B))) (A)] [                       A   ]
```

The B subforms that remain each reduce via Pervasion, Involution, and Occlusion:

```
      [                  (A)] [                       A   ]
      [                   A ] [                     [ A ] ]
      [                   A ] [                     [ A ] ]
      [                  (A)] [                       A   ]
```

Each remaining line, without B subforms, is identical.

Analysis

Working backwards, the four varieties of Kauffman's single axiom are all formed from the same base:

```
            ( )  ==>  (A) ( )  ==>  (A) ((A))
```

The interior spaces of each A subform are enriched by three different void-equivalent B subforms:

```
        I.  (( ))  ==>  ( B ( ))

       II.  (( ))  ==>  ((B)( ))

      III.  (( ))  ==>  ((B)( ))  ==>  ((B)((B)))
```

The pattern is

```
            1           [A   III ] [[A]  X ]
            2           [A  I  II] [[A] III]
            3           [A   III ] [[A]  X ]
            4           [A  I  II] [[A] III]
```

X stands in place of the subforms that are subsumed, without an intermediate reduction step.

From this pattern, we see that varieties 1 and 3 are still identical, as are varieties 2 and 4.

```
            1            ( A  B)( A (B))  = (A)
            2            ((A) B)((A)(B))  =  A
            3           (( A  B)( A (B))) =  A
            4           (((A) B)((A)(B))) = (A)
```

This highlights the inversion of each through bounding each side of the equations.

What is interesting is that the Robbins Problem highlights the differences between pair 1 and 2, which are subject to the Robbins question, and pair 3 and 4, which are conventional Boolean.

That is to say, the differences that the Robbins Problem highlight continue *not* to show up as relevant to Brownian analysis.