

THE CANONS OF FORMAL SYMBOL SYSTEMS

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The canons of formal symbol systems are those conventions that guide our agreement about the semantics, or meaning, and the syntax, or representation, of formal systems. These canons define the meaning of formality. An error within a formal system is defined as a violation of one of these canons. The canons provide a complete set of rules to motivate mathematical behavior.

The following list is loosely paraphrased from Spencer-Brown's *Laws of Form*, one of the few mathematics texts that addresses the assumptions underlying the use of formal symbol systems.

CANONS

INTENTION

- Permissible transformations are specified by rules.
- New rules or conventions must be mutually agreed upon.
- What is not explicitly permitted is forbidden.
- Formal constraints are by choice.

REFERENCE

- The representation of a concept is not inherently confusing.
- Symbols represent concepts that can be understood.
- **Contraction of reference:** Representations may be abbreviated to any extent that is understood.
- **Expansion of reference:** New representations may be introduced without limit, provided that confusion is avoided.

SUBSTITUTION

- In any expression, any arrangement can be exchanged for an equivalent arrangement.
- Equivalent intentions are expressed by equivalent symbols.

SIMPLIFICATION/TRANSFORMATION

- Following the rules does not change the meanings.
- Meaning is preserved when steps are taken.
- Symbols may be manipulated within the rules without altering their assigned interpretations.

RELEVANCE

- Universal properties within a system need not be indicated.
- A mutually agreed upon interpretative context is assumed.
- If a property is common to every indication, it need not be indicated.

DEMONSTRATION

- A demonstration rests in a finite number of steps.

DOMINANCE

- Meaning is indicated overtly in the form of expressions.
- The form of an expression indicates the choice of a meaning.

TRANSMISSION

- When a value of a variable oscillates (by ambiguity or by choice), the interpretative context of the variable is either transparent or opaque to that oscillation.

AXIOMS and THEOREMS of REPRESENTATION

AXIOMS

- The interpretation of a symbol does not change independently of the interpreter.
- Reinterpretation denies the original interpretation.

EQUIVALENCE OF TRANSFORMATION

- The product of transformation steps is independent of the choice or sequence of steps, given a consistent result.
- All transformation paths leading to the same result are equivalent.

THEOREMS OF REPRESENTATION

- **Form:** An expression is represented by its form.
- **Content:** The value of an expression is the value of its form.
- **Agreement:** Simplification is unique.
- **Distinction:** Rules specify permissible transformation steps.
- Steps do not change the meaning of an expression, although they do change the form.
- If the value of two expressions differ, then steps taken on the two expressions maintain that difference.

PROCEDURAL THEOREMS

- **Identity:** Expressions with identical forms represent identical values.
- **Value:** Expressions with the same value can be identified.
- **Consequence:** Expressions equivalent in value to a given expression are equivalent to each other.

CONNECTIVE THEOREMS

- **Invariance:** If a distinction distinguishes equivalent expressions, it is not a distinction.
- **Variance:** If different expressions each contain the same arrangement, then this arrangement can be communicated to the interpretative context of the expressions.